# Formulário de estudo de EAC – 2015.v1

## Algoritmo de cálculo

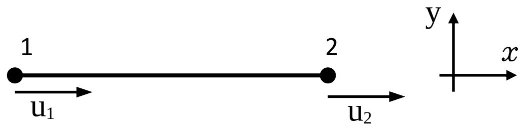
1 - Discretizar o problema
2 - Identificar os graus de liberdade e condições de fronteira
3 - Escolher o(s) elemento(s) finito(s) mais adequado a cada discretização
4 - Escrever a matriz de conectividades dos elementos
5 - Determinar a matriz de rigidez global / simplificada do problema
6 - Determinar o vetor de forças
7 - Resolver o sistema de equações lineares para obter deslocamentos e/ou forças
8 - Com o campo de descolamentos podemos determinar o campo das extensões e o campo das tensões
9 - Para verificar os cálculos pode confirmar o equilíbrio de forças e/ou o equilíbrio energético.

$$\sum F_{x,y} = 0; \sum M_0 = 0 \qquad \sum_i \boldsymbol{U}_e^{(i)} = \frac{1}{2}\sum_j W_j \qquad \boldsymbol{U}_e = \frac{1}{2}\int_V \sigma_x \varepsilon_x \, dV$$

$$c_1 = \cos(\alpha_1) \quad c_1^2 = \cos^2(\alpha_1) \quad s_1 = \sin(\alpha_1) \quad s_1^2 = \sin^2(\alpha_1)$$

$$c_2 = \cos(\alpha_2) \quad c_2^2 = \cos^2(\alpha_2) \quad s_2 = \sin(\alpha_2) \quad s_2^2 = \sin^2(\alpha_2)$$

## EB2GL - Elemento Barra (Cargas: axiais)

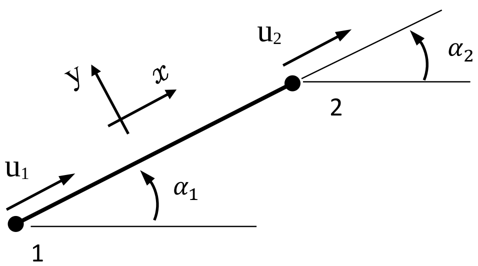


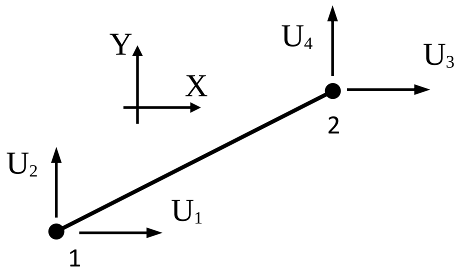

$$\sigma_x = E\frac{(u_2 - u_1)}{L}$$

$$\boxed{f_T = \begin{bmatrix} -\alpha EA\Delta \mathrm{T} & \alpha EA\Delta \mathrm{T} \end{bmatrix}^T}$$

$$u(x) = \begin{bmatrix} N \end{bmatrix}_B \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \qquad \boxed{\begin{bmatrix} N \end{bmatrix}_B = \begin{bmatrix} 1 - \dfrac{x}{L} & \dfrac{x}{L} \end{bmatrix}} \qquad \boxed{\begin{bmatrix} K \end{bmatrix}_B = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}} \qquad \boxed{\begin{bmatrix} M \end{bmatrix}_B = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}$$

## EB2GLR - Elemento Barra Rodado (Cargas: axiais)



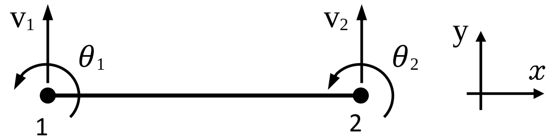

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} R \end{bmatrix}_B \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{Bmatrix}$$

$$\boxed{\begin{bmatrix} R \end{bmatrix}_B = \begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & c_2 & s_2 \end{bmatrix}} \qquad \begin{bmatrix} K \end{bmatrix}_B^R = \begin{bmatrix} R \end{bmatrix}_B^T \begin{bmatrix} K \end{bmatrix}_B \begin{bmatrix} R \end{bmatrix}_B \qquad \begin{bmatrix} M \end{bmatrix}_B^R = \begin{bmatrix} R \end{bmatrix}_B^T \begin{bmatrix} M \end{bmatrix}_B \begin{bmatrix} R \end{bmatrix}_B \qquad \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix} = \begin{bmatrix} R \end{bmatrix}_B^T \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

$$\boxed{\begin{bmatrix} K \end{bmatrix}_B^R = \frac{EA}{L}\begin{bmatrix} c_1^2 & s_1c_1 & -c_1c_2 & -s_2c_1 \\ s_1c_1 & s_1^2 & -s_1c_2 & -s_1s_2 \\ -c_1c_2 & -s_1c_2 & c_2^2 & s_2c_2 \\ -s_2c_1 & -s_1s_2 & s_2c_2 & s_2^2 \end{bmatrix}} \qquad \boxed{\begin{bmatrix} M \end{bmatrix}_B^R = \frac{\rho AL}{420}\begin{bmatrix} 140c_1^2 & 140s_1c_1 & 70c_1c_2 & 70s_2c_1 \\ 140s_1c_1 & 140s_1^2 & 70s_1c_2 & 70s_1s_2 \\ 70c_1c_2 & 70s_1c_2 & 140c_2^2 & 140s_2c_2 \\ 70s_2c_1 & 70s_1s_2 & 140s_2c_2 & 140s_2^2 \end{bmatrix}}$$

# EV4GL - Elemento de Viga

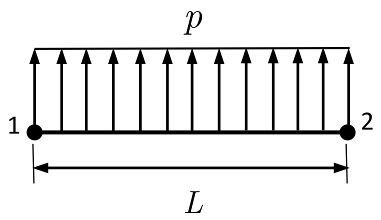
(Cargas: transversais e momentos fletores)



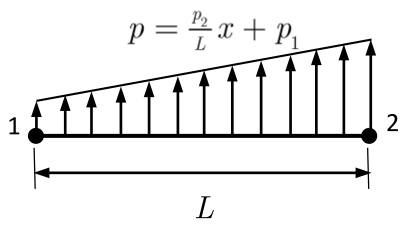

$$V(x) = \frac{dM(x)}{dx} \qquad M(x) = EI_z \frac{d^2 v(x)}{dx^2} \qquad \sigma_x(x,y) = -\frac{M(x)\cdot y}{I_z}$$

$$\boxed{[K]_V = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}} \qquad \boxed{[M]_V = \frac{\rho A L}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}}$$

$$v(x) = [N]_V \begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} \qquad \boxed{[N]_V = \begin{bmatrix} 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} & x - \frac{2x^2}{L} + \frac{x^3}{L^2} & \frac{3x^2}{L^2} - \frac{2x^3}{L^3} & -\frac{x^2}{L} + \frac{x^3}{L^2} \end{bmatrix}}$$



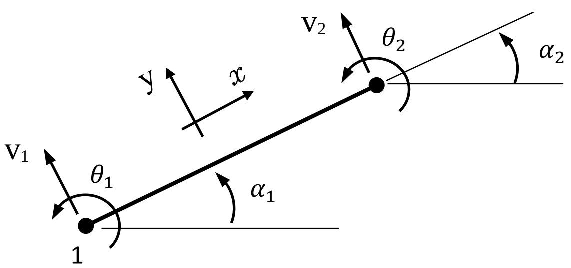

$$\boxed{f_V = \begin{bmatrix} \frac{pL}{2} & \frac{pL^2}{12} & \frac{pL}{2} & -\frac{pL^2}{12} \end{bmatrix}^T}$$

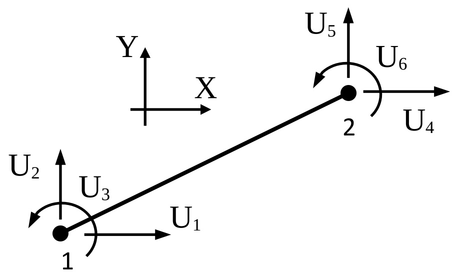



$$\boxed{f_V = \begin{bmatrix} \frac{3p_2L}{20} + \frac{p_1L}{2} & \frac{p_2L^2}{30} + \frac{p_1L^2}{12} & \frac{7p_2L}{20} + \frac{p_1L}{2} & -\frac{p_2L^2}{20} - \frac{p_1L^2}{12} \end{bmatrix}^T}$$

# EV4GLR - Elemento de Viga Rodado

(Cargas: transversais e momentos fletores)



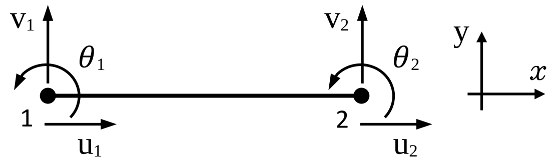

$$\begin{Bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{Bmatrix} = [R]_V \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}$$

$$[R]_V = \begin{bmatrix} -s_1 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s_2 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad [K]_V^R = \frac{EI}{L^3}\begin{bmatrix} 12s_1^2 & -12s_1c_1 & -6Ls_1 & -12s_1s_2 & -12s_1c_2 & -6Ls_1 \\ -12s_1c_1 & 12c_1^2 & 6Lc_1 & 12c_1s_2 & -12c_1c_2 & 6Lc_1 \\ -6Ls_1 & 6Lc_1 & 4L^2 & 6Ls_2 & -6Lc_2 & 2L^2 \\ -12s_1s_2 & 12c_1s_2 & 6Ls_2 & 12s_2^2 & -12s_1c_2 & 6Ls_2 \\ -12s_1c_2 & 6Ls_2 & -6Lc_2 & -12s_1c_2 & 12c_2^2 & -6Lc_2 \\ -6Ls_1 & 6Lc_1 & 2L^2 & 6Ls_2 & -6Lc_2 & 4L^2 \end{bmatrix}$$

$$[K]_V^R = [R]_V^T [K]_V [R]_V \qquad F_V = [R]_V^T \; f_V$$

# EVB6GL - Elemento de Viga Barra

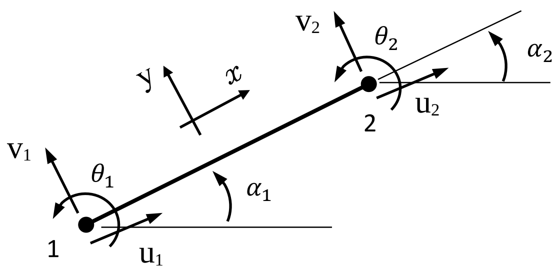
(Cargas: axiais, transversais e momentos fletores)

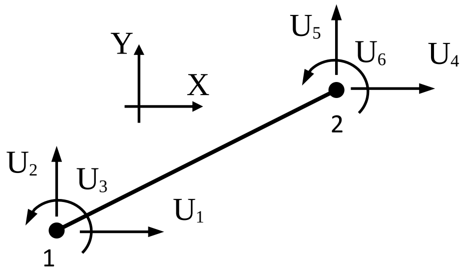



$$[K]_{VB} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

# EV6GLR - Elemento de Viga Barra Rodado

(Cargas: axiais, transversais e momentos fletores)




$$\begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \end{Bmatrix} = [R]_{VB} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix}$$

$$\boxed{[R]_{VB} = \begin{bmatrix} c_1 & s_1 & 0 & 0 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 & s_2 & 0 \\ 0 & 0 & 0 & -s_2 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$[K]^R_{VB} = [R]^T_{VB} [K]_{VB} [R]_{VB}$$

$$F_{VB} = [R]^T_{VB} \; f_{VB}$$

$$\boxed{[K]^R_{VB} = E \begin{bmatrix} \frac{Ac_1^2}{L} + \frac{12Is_1^2}{L^3} & \frac{Ac_1s_1}{L} - \frac{12Is_1c_1}{L^3} & -\frac{6Is_1}{L^2} & -\frac{Ac_1c_2}{L} - \frac{12Is_1s_2}{L^3} & -\frac{Ac_1s_2}{L} + \frac{12Is_1c_2}{L^3} & -\frac{6Is_1}{L^2} \\ \frac{Ac_1s_1}{L} - \frac{12s_1c_1}{L^3} & \frac{As_1^2}{L} + \frac{12Ic_1^2}{L^3} & \frac{6Ic_1}{L^2} & -\frac{As_1c_2}{L} + \frac{12Ic_1s_2}{L^3} & -\frac{As_1s_2}{L} - \frac{12Ic_1c_2}{L^3} & \frac{6Ic_1}{L^2} \\ -\frac{6Is_1}{L^2} & \frac{6Ic_1}{L^2} & \frac{4I}{L} & \frac{6Is_2}{L^2} & -\frac{6Ic_2}{L^2} & \frac{2I}{L} \\ -\frac{Ac_1c_2}{L} - \frac{12Is_1s_2}{L^3} & -\frac{As_1c_2}{L} + \frac{12Ic_1s_2}{L^3} & \frac{6Is_2}{L^2} & \frac{Ac_2^2}{L} + \frac{12Is_2^2}{L^3} & \frac{Ac_2s_2}{L} - \frac{12Is_2c_2}{L^3} & \frac{6Is_2}{L^2} \\ -\frac{Ac_1s_2}{L} + \frac{12Is_1c_2}{L^3} & -\frac{As_1s_2}{L} - \frac{12Ic_1c_2}{L^3} & -\frac{6Ic_2}{L^2} & \frac{Ac_2s_2}{L} - \frac{12Is_2c_2}{L^3} & \frac{As_2^2}{L} + \frac{12Ic_2^2}{L^3} & -\frac{6Ic_2}{L^2} \\ -\frac{6Is_1}{L^2} & \frac{6Ic_1}{L^2} & \frac{2I}{L} & \frac{6Is_2}{L^2} & -\frac{6Ic_2}{L^2} & \frac{4I}{L} \end{bmatrix}}$$