ESTIMATION OF UNMEASURED FREQUENCY RESPONSE FUNCTIONS

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In the present work, the authors propose a new methodology for the estimation of Frequency Response Functions (*FRFs*) that for some reason cannot be measured in a structure. With such an objective, one uses *FRFs* that are obtained through a finite element modelization of the structure. Due to instrumentation limitations, in the majority of the cases the *FRFs* that are associated to rotational degrees-of-freedom are often problematic. In the examples that are given in this paper those kind of *FRFs* are the ones to be estimated. Various examples are numerically simulated and the results obtained are discussed in order to assess the robustness of the method to the presence of noise in the data.

1. Introduction

In most situations the determination of *FRFs* in complex structures is made in zones geometrically well defined, often areas that are easy to model numerically. However, experimentally, some of those *FRFs* are difficult to measure. Assuming that part of the structure has been modeled numerically, for instance using finite elements, various authors have developed expressions that allow the evaluation of the *FRFs* that are difficult or impossible to measure, namely those involving rotational degrees-of-freedom.

Yang and Park[1], starting from the product of the dynamic stiffness matrix of the structure with the respective *FRF* matrix, deduced an expression for estimating those *FRFs* that are difficult to measure. Celic and Boltežar[2] and Wang *et al*[3] used that expression, but the results were not particularly good when the data were subjected to some level of noise.

Avitabile and O'Callahan[4] used the SEREP method to undertake an expansion process and thus estimate the rotations that had not been measured; however, they did not evaluate the sensitivity of the procedure to any possible perturbation.

Silva *et al*[5] proposed a method for estimating rotational *FRFs*, based on the canceling of multiple additional masses, using the classical *FRF* uncoupling algorithm; they concluded that the results were very good for perfect data, but extremely sensitive to noise in the experimental data.

In this paper a general formulation is developed to estimate unmeasured *FRFs*. Several numerical simulations are carried out and the results are discussed, when varying the number of measured translational *FRFs* and when adding some noise to simulate experimental data. The final expressions are very similar to those of Wang *et al*[3].

2. Theoretical Formulation

The dynamic characterization of structures can be made based upon mass, stiffness and damping matrices (M, K, C), defining the correspondent dynamic stiffness Z or based on measured *FRFs* through the receptance matrix H. The formalism used in this work is mainly focused on the knowledge of *FRFs*, either obtained numerically from the dynamic stiffness Z or experimentally measured, namely at translational degrees-of-freedom; the objective is to estimate the remaining unmeasured *FRFs*.

Let us consider two substructures A and B, connected through some co-ordinates as shown in Fig. 1. Together, they form structure C. Substructure A is supposed to be easy to model numerically using finite element modeling (*FEM*); the internal co-ordinates are designated by the letter i and the co-ordinates common to substructure B are called j; these are generally inaccessible or unmeasured.

Co-ordinates i are divided into two groups: experimentally measured co-ordinates t and co-ordinates r, which – for some reason – have not been measured.
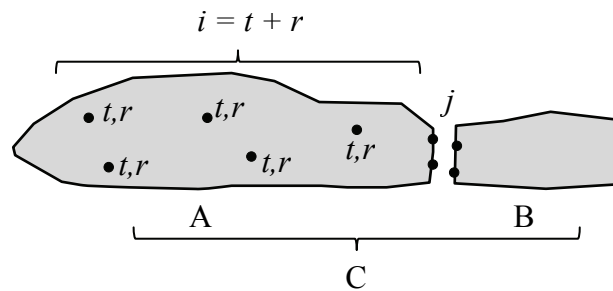

Figure 1. Coupling of substructures A and B, forming structure C.

The objective is therefore to evaluate the *FRFs* at co-ordinates r and j of structure C, knowing the information obtained numerically from substructure A and the measurements at co-ordinates t of structure C.

The dynamic stiffness of structure C, $\boldsymbol{Z}^{C}$, can be written as the sum of the dynamic stiffness of substructure A, $\boldsymbol{Z}^{A}$, with the dynamic stiffness of substructure B, $\boldsymbol{Z}^{B}$, having into account the right connection co-ordinates:

$$\boldsymbol{Z}^{C} = \boldsymbol{Z}^{A} + \boldsymbol{Z}^{B} \tag{1}$$

Rearranging the second member of Eq. (1),

$$\boldsymbol{Z}^{C} = \boldsymbol{Z}^{A}\left(\boldsymbol{I} + \boldsymbol{H}^{A}\boldsymbol{Z}^{B}\right) \tag{2}$$

where $\boldsymbol{H}^{A}$ is the *FRF* matrix of A, i.e., the inverse of the dynamic stiffness $\boldsymbol{Z}^{A}$. Inverting both members,

$$\boldsymbol{H}^{C} = \left(\boldsymbol{I} + \boldsymbol{H}^{A}\boldsymbol{Z}^{B}\right)^{-1}\boldsymbol{H}^{A} \tag{3}$$

Explicitly, in terms of the sub-matrices involving co-ordinates i and j,

$$\begin{bmatrix} \boldsymbol{H}^{C}_{ii} & \boldsymbol{H}^{C}_{ij} \\ \boldsymbol{H}^{C}_{ji} & \boldsymbol{H}^{C}_{jj} \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I}_{ii} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{jj} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}^{A}_{ii} & \boldsymbol{H}^{A}_{ij} \\ \boldsymbol{H}^{A}_{ji} & \boldsymbol{H}^{A}_{jj} \end{bmatrix}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}^{B}_{jj} \end{bmatrix}\right]^{-1}\begin{bmatrix} \boldsymbol{H}^{A}_{ii} & \boldsymbol{H}^{A}_{ij} \\ \boldsymbol{H}^{A}_{ji} & \boldsymbol{H}^{A}_{jj} \end{bmatrix} \tag{4}$$

Simplifying Eq. (4), it follows that

$$\begin{bmatrix} \boldsymbol{H}^{C}_{ii} & \boldsymbol{H}^{C}_{ij} \\ \boldsymbol{H}^{C}_{ji} & \boldsymbol{H}^{C}_{jj} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{ii} & -\boldsymbol{H}^{A}_{ij}\boldsymbol{Z}^{B}_{jj}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}^{A}_{jj}\boldsymbol{Z}^{B}_{jj}\right)^{-1} \\ \boldsymbol{0} & \left(\boldsymbol{I}_{jj} + \boldsymbol{H}^{A}_{jj}\boldsymbol{Z}^{B}_{jj}\right)^{-1} \end{bmatrix}\begin{bmatrix} \boldsymbol{H}^{A}_{ii} & \boldsymbol{H}^{A}_{ij} \\ \boldsymbol{H}^{A}_{ji} & \boldsymbol{H}^{A}_{jj} \end{bmatrix} \tag{5}$$

from which,

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A} \\ \left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A} & \left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (6)$$

To facilitate the development of the formulation it is important to note that

$$\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1} = \boldsymbol{Z}_{jj}^{B}\left[\left(\boldsymbol{Z}_{jj}^{B}\right)^{-1}\left(\boldsymbol{Z}_{jj}^{B} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)\right]^{-1}$$
$$= \boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{Z}_{jj}^{B} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{Z}_{jj}^{B} = \boldsymbol{Z}_{jj}^{B}\left(\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{Z}_{jj}^{B} = \left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B} \quad (7)$$

Equation (4) can also be written as

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{jj}^{B} \end{bmatrix}\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (8)$$

or simply

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \\ \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \end{bmatrix} \quad (9)$$

Let us take the following expressions from Eqs. (6), (7) and (9):

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (10)$$

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} \quad (11)$$

Pre-multiplying Eqs. (10) and (11) by $\left(\boldsymbol{H}_{ij}^{A}\right)^{+}$, leads to

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (12)$$

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} \quad (13)$$

Pre-multiplying Eq. (12) by $\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)$,

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (14)$$

Rearranging Eq. (14),

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\left[\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right)\right] \quad (15)$$

Substituting Eq. (15) in Eq. (13), one obtains $\boldsymbol{H}_{ji}^{C}$ regardless of the dynamic stiffness of B.

$$\boldsymbol{H}_{ji}^{C} = \boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) \quad (16)$$

An entirely similar process can be used to determine $\boldsymbol{H}_{jj}^{C}$, regardless of the dynamic stiffness of B; once again, from Eq. (6), Eq. (7) and Eq. (9), one has

$$\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A} \quad (17)$$

$$\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \quad (18)$$

Pre-multiplying Eqs. (17) and (18) by $\left(H_{ij}^{A}\right)^{+}$, leads to

$$\left(H_{ij}^{A}\right)^{+}\left(H_{ij}^{A}-H_{ij}^{C}\right)=\left(I_{jj}+Z_{jj}^{B}H_{jj}^{A}\right)^{-1}Z_{jj}^{B}H_{jj}^{A} \tag{19}$$

$$\left(H_{ij}^{A}\right)^{+}\left(H_{ij}^{A}-H_{ij}^{C}\right)=Z_{jj}^{B}H_{jj}^{C} \tag{20}$$

Pre-multiplying Eq. (19) by $\left(I_{jj}+Z_{jj}^{B}H_{jj}^{A}\right)$,

$$\left(H_{ij}^{A}\right)^{+}\left(H_{ij}^{A}-H_{ij}^{C}\right)+Z_{jj}^{B}H_{jj}^{A}\left(H_{ij}^{A}\right)^{+}\left(H_{ij}^{A}-H_{ij}^{C}\right)=Z_{jj}^{B}H_{jj}^{A} \tag{21}$$

which simplifies to

$$\left(H_{ij}^{A}\right)^{+}\left(H_{ij}^{A}-H_{ij}^{C}\right)=Z_{jj}^{B}H_{jj}^{A}\left(H_{ij}^{A}\right)^{+}H_{ij}^{C} \tag{22}$$

Substituting Eq. (22) in Eq. (20), one obtains H_{jj}^{C} regardless of the dynamic stiffness of B.

$$H_{jj}^{C}=H_{jj}^{A}\left(H_{ij}^{A}\right)^{+}H_{ij}^{C} \tag{23}$$

Finally, to determine H_{ii}^{C} regardless of the dynamic stiffness of B, one starts by rearranging Eq. (17):

$$\left(H_{ij}^{A}-H_{ij}^{C}\right)\left(H_{jj}^{A}\right)^{-1}=H_{ij}^{A}\left(I_{jj}+Z_{jj}^{B}H_{jj}^{A}\right)^{-1}Z_{jj}^{B} \tag{24}$$

Substituting Eq. (24) in Eq. (10) leads to

$$H_{ii}^{A}-H_{ii}^{C}=\left(H_{ij}^{A}-H_{ij}^{C}\right)\left(H_{jj}^{A}\right)^{-1}H_{ji}^{A} \tag{25}$$

Transposing (25), one finally obtains

$$H_{ii}^{C}=H_{ii}^{A}-H_{ij}^{A}\left(H_{jj}^{A}\right)^{-1}\left(H_{ji}^{A}-H_{ji}^{C}\right) \tag{26}$$

2.1 Summary

One can now determine H^{C} without needing to know anything about substructure B, using the following equations:

$$H_{ji}^{C}=H_{ji}^{A}-H_{jj}^{A}\left(H_{ij}^{A}\right)^{+}\left(H_{ii}^{A}-H_{ii}^{C}\right) \tag{16}$$

$$H_{ii}^{C}=H_{ii}^{A}-H_{ij}^{A}\left(H_{jj}^{A}\right)^{-1}\left(H_{ji}^{A}-H_{ji}^{C}\right) \tag{26}$$

$$H_{jj}^{C}=H_{jj}^{A}\left(H_{ij}^{A}\right)^{+}H_{ij}^{C} \tag{23}$$

2.2 Estimation of unmeasured *FRFs*

Co-ordinates i of structure C are formed by co-ordinates t and r. Matrix H^{C}, symmetric, is constructed as follows:

$$H^{C}=\begin{bmatrix}H_{ii}^{C} & H_{ij}^{C}\\ H_{ji}^{C} & H_{jj}^{C}\end{bmatrix}\qquad i=t+r\quad\rightarrow\quad H^{C}=\begin{bmatrix}H_{tt}^{C} & H_{tr}^{C} & H_{tj}^{C}\\ H_{rt}^{C} & H_{rr}^{C} & H_{rj}^{C}\\ H_{jt}^{C} & H_{jr}^{C} & H_{jj}^{C}\end{bmatrix} \tag{27}$$

One supposes that in matrix $\boldsymbol{H}^C$ it is only possible to determine the sub-matrix $\boldsymbol{H}^C_{tt}$ experimentally. With this information, the remainder of the array elements are determined using Eqs. (16, 26 e 23).

From Eq. (16), considering only translational co-ordinates, i.e., $ii = tt$, one has

$$\boldsymbol{H}^C_{jt} = \boldsymbol{H}^A_{jt} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\left(\boldsymbol{H}^A_{tt} - \boldsymbol{H}^C_{tt}\right) \tag{28}$$

From Eqs. (26) and (28), relating rotational to translational co-ordinates such that $ii = rt$, it follows that

$$\boldsymbol{H}^C_{rt} = \boldsymbol{H}^A_{rt} - \boldsymbol{H}^A_{rj}\left(\boldsymbol{H}^A_{jj}\right)^{-1}\left(\boldsymbol{H}^A_{jt} - \boldsymbol{H}^C_{jt}\right) \tag{29}$$

From Eqs. (16) and (29), relating translational to rotational co-ordinates such that $ii = tr$, the result is

$$\boldsymbol{H}^C_{jr} = \boldsymbol{H}^A_{jr} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\left(\boldsymbol{H}^A_{tr} - \boldsymbol{H}^C_{tr}\right) \tag{30}$$

From Eqs. (26) and (30), considering only rotational co-ordinates, i.e., $ii = rr$, one has,

$$\boldsymbol{H}^C_{rr} = \boldsymbol{H}^A_{rr} - \boldsymbol{H}^A_{rj}\left(\boldsymbol{H}^A_{jj}\right)^{-1}\left(\boldsymbol{H}^A_{jr} - \boldsymbol{H}^C_{jr}\right) \tag{31}$$

Finally, Eqs. (23) and (28), for $i = t$, yield

$$\boldsymbol{H}^C_{jj} = \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\boldsymbol{H}^C_{tj} \tag{32}$$

All the elements of matrix $\boldsymbol{H}^A$ are determined numerically by *FEM*. Thus, in sequence, starting from the known responses measured experimentally ($\boldsymbol{H}^C_{tt}$), it is possible to estimate all the responses in the other co-ordinates r and j of structure C.

2.3 Frequency-domain correlation criterion

A simple visual comparison of the *FRFs* calculated numerically ($H_A(\omega)$) with those obtained experimentally ($H_X(\omega)$) only provides a qualitative idea of the goodness of the results. It is important to have a quantitative result. Various correlation indices have been proposed along the years to compare *FRFs*. In our case one has selected the Local Amplitude Criterion, *LAC*, defined as [6]:

$$LAC_{ij}(\omega) = \frac{2\left|H_{Xij}(\omega)^* \cdot H_{Aij}(\omega)\right|}{\left(H_{Xij}(\omega)^* \cdot H_{Xij}(\omega)\right) + \left(H_{Aij}(\omega)^* \cdot H_{Aij}(\omega)\right)} \tag{33}$$

where i and j are the response and excitation co-ordinates, respectively, $H_{Aij}(\omega)$ is the *FRF* obtained numerically and $H_{Xij}(\omega)$ is the *FRF* obtained experimentally, both at frequency ω; * denotes complex conjugate. The correlation index varies between 0 and 1 for each frequency ω. $H_{Xij}(\omega)$ is closely related to $H_{Aij}(\omega)$ if $LAC_{ij}(\omega)$ has all its values close to 1. One can determine an average result and thus have a unique value quantifying the correlation between both *FRFs*:

$$\overline{LAC_{ij}} = \frac{1}{N}\sum_{k=1}^{N} LAC_{ij}(\omega_k) \tag{34}$$

This index will be used to analyse the results of the case studies that follow in the next section.

3. Simulation studies

To validate the proposed method, four numerical examples of coupled structures are presented and illustrated in Figs. 2, 3, 4 and 5. Each coupled structure is constituted by three components, A_1, B and A_2, forming structure C. A_1 and A_2 form substructure A.
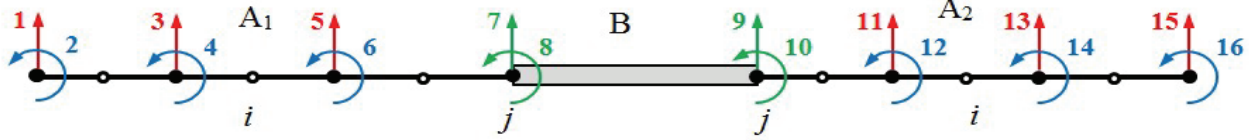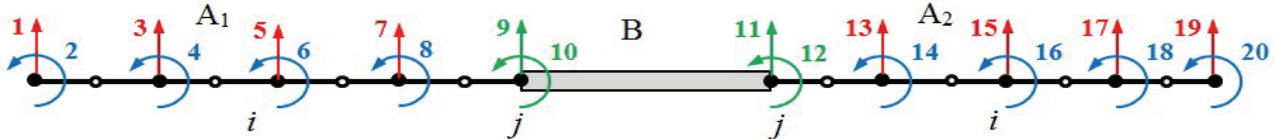

Figure 2. Case 1.

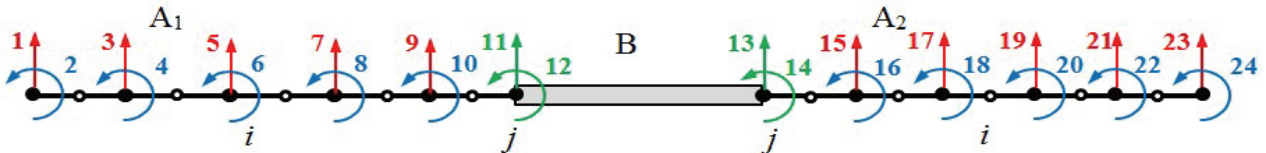

Figure 3. Case 2.

Figure 4. Case 3.

Figure 5. Case 4.

Finite beam elements, each one with four degrees of freedom are used; each component A_1 and A_2 is divided into various elements. Only the nodes indicated in the figures are to be considered as co-ordinates for our study. In Figs. 2, 3, 4 and 5, the translational co-ordinates t (known) are the ones in red colour; in blue, one has the rotational co-ordinates r (unknown); the joint co-ordinates j (unknown) are represented in green. The characteristics of each component are displayed in Table 1.

Table 1. Characteristics of the components of the beam.

Beam	Length	Width	Thickness	E	ρ
A_1	270 mm	30 mm	5 mm	194 GPa	7562 Kg/m^3
B	200 mm	30 mm	10 mm	194 GPa	7562 Kg/m^3
A_2	370 mm	30 mm	5 mm	194 GPa	7562 Kg/m^3

3.1 Numerical noise

To simulate the experimental errors of $\boldsymbol{H}_{tt}^{C}$, a numerical random error independent of the amplitude will be added and defined as follows[7]:

$$\tilde{H}_{tt}(\omega_k) = H_{tt}(\omega_k) + \frac{\gamma}{100} \cdot normrnd(\omega_k) \cdot \max\left(\left\|H_{tt}(\omega)\right\|\right) \tag{35}$$

where γ is the noise level in percentage and $normrnd(\omega)$ is a normal distribution with zero mean and standard deviation equal to one. A noise level of 3% has been added.

3.2 Quantification of the effect of the error

The correlation criterion presented in Section 2.3 will be used to show the influence of the number of chosen co-ordinates i. Figures 6, 7, 8 and 9 show the averaged *LAC* for each matrix $\boldsymbol{H}^C$. In each figure, the darker and larger the results, the better the correlation.
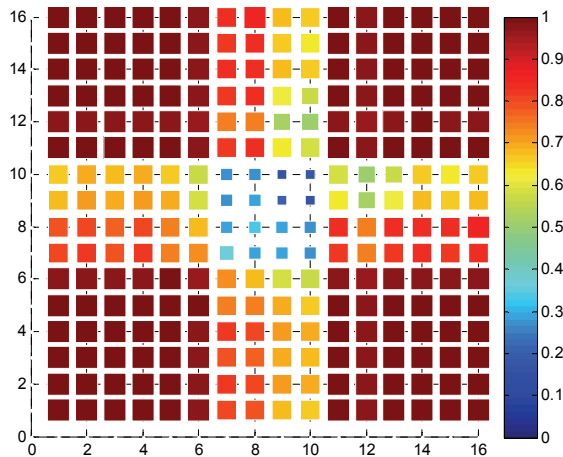
Figure 6. Averaged *LAC* - Case 1**.**
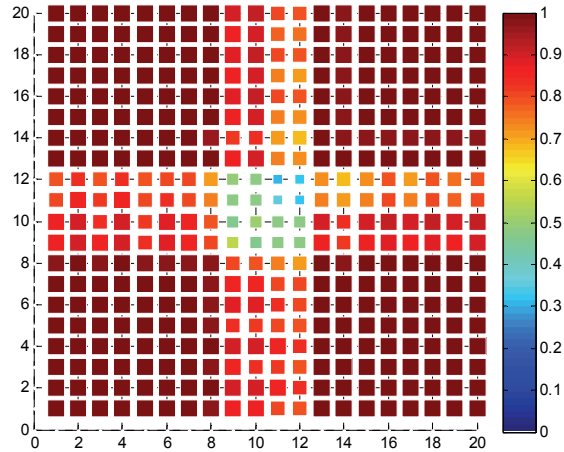
Figure 7. Averaged *LAC* - Case 2**.**
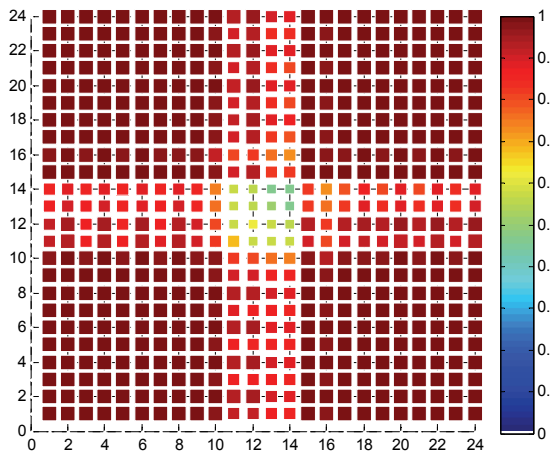
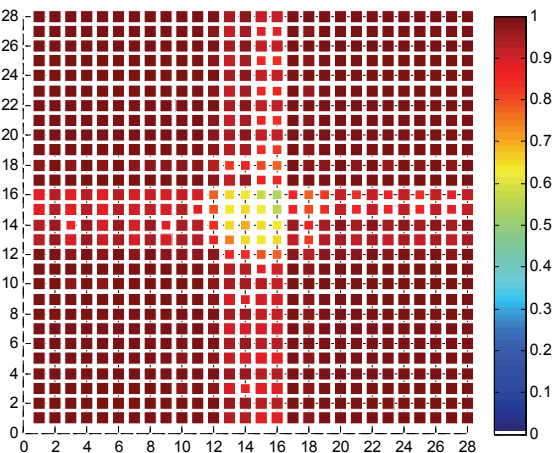**Figure 8**. Averaged *LAC* - Case 3.

Figure 9. Averaged *LAC* - Case 4.

The values relating co-ordinates i show a very good correlation. However, the results involving co-ordinates j show a poor correlation. This effect is more significant for correlations between the co-ordinates j themselves.

There are considerable improvements if the number of co-ordinates i to be measured increases, as it can be observed when comparing Fig. 6 with Fig. 9.

Figures 10 and 11 present two of the estimated *FRFs* of Case 1 against their theoretical values. These *FRFs* relate rotation due to a moment ($\boldsymbol{H}^C_{24}$) and displacement due to a moment ($\boldsymbol{H}^C_{36}$). Only some disturbances are noticed at the anti-resonances.
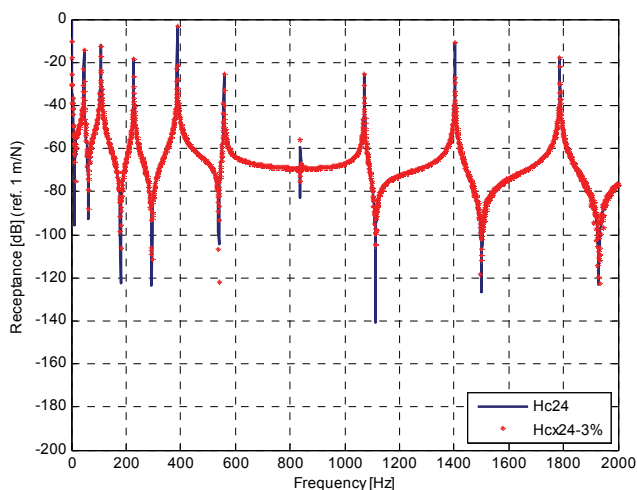
Figure 10. H_{24}^C - Case 1.
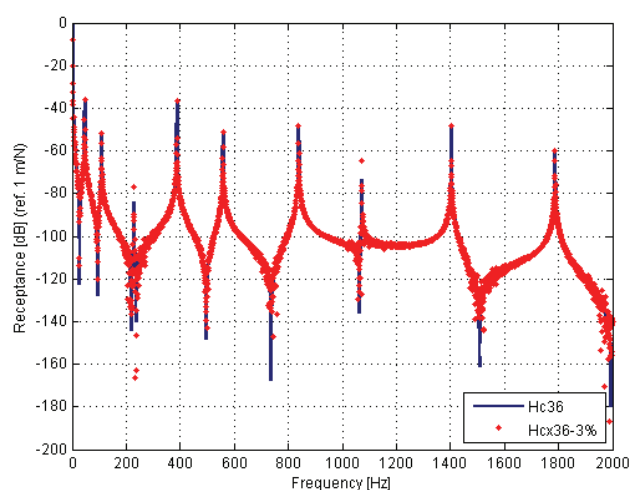
Figure 11. H_{36}^C - Case 1.

4. Conclusions

In this paper a new methodology for estimating unmeasured *FRFs* has been presented, based upon classical formulations of *FRF* coupling and uncoupling of substructures; the process implies the knowledge of an analytical or numerical model of part of the structure and the measurement of translational *FRFs*. This methodology can be used to estimate *FRFs* involving rotational degrees-of-freedom, often difficult or impossible to measure.

Some numerical examples have been presented to evaluate the performance of the technique and some noise has been added to simulate real data. Not surprisingly, it has been shown that an increase in the number of measured translational degrees-of-freedom improves the estimation of the unmeasured *FRFs*. The results are very promising for future applications in real experimental cases.

Acknowledgements

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