INDEX

TRIGONOMETRIC FUNCTIONS

a) Relations between the functions of the same angle

$\text{sen}^2\ \alpha + \cos^2 \alpha = 1$

$\text{sen}\ \alpha = \sqrt{1 - \cos^2 \alpha} = \text{tg}\ \alpha \ / \ \sqrt{1 + \text{tg}^2\ \alpha}$

$\cos \alpha = \sqrt{1 - \text{sen}^2\ \alpha} = 1 \ / \ \sqrt{1 + \text{tg}^2\ \alpha}$

$\text{tg}\ \alpha = \text{sen}\ \alpha / \cos \alpha$

$\text{ctg}\ \alpha = \cos \alpha / \text{sen}\ \alpha = 1/\text{tg}\ \alpha$

$\text{tg}\ \alpha = \text{sen}\ \alpha \ / \ \sqrt{1 - \text{sen}^2\ \alpha}$

$\sec \alpha = 1/\cos \alpha$

$\text{cosec}\ \alpha = 1/\text{sen}\ \alpha$

b) Relations between the functions of two angles

$\text{sen}\ (\alpha \pm \beta) = \text{sen}\ \alpha \cos \beta \pm \cos \alpha\ \text{sen}\ \beta$

$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \pm \text{sen}\ \alpha\ \text{sen}\ \beta$

$\text{tg}\ (\alpha \pm \beta) = (\text{tg}\ \alpha \pm \text{tg}\ \beta) \ / \ (1 \mp \text{tg}\ \alpha\ \text{tg}\ \beta)$

c) Multiples and sub multiples of an angle

$\text{sen}\ 2\ \alpha = 2\ \text{sen}\ \alpha \cos \alpha$

$\cos 2\ \alpha = \cos^2 \alpha - \text{sen}^2\ \alpha = 2 \cos^2 \alpha - 1$

$\text{tg}\ 2\ \alpha = 2\ \text{tg}\ \alpha \ / \ (1 - \text{tg}^2\ \alpha)$

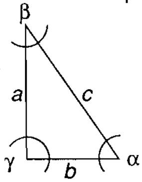
$\text{sen}\ (\alpha/2) = \sqrt{(1 - \cos \alpha)/2}$

$\cos (\alpha/2) = \sqrt{(1 + \cos \alpha)/2}$

$\text{tg}\ (\alpha/2) = \text{sen}\ \alpha/(1 + \cos \alpha).$

MAIN THEOREMS ON TRIANGLES

A) Right-angled triangle (*a* and *b* catheti, *c* hypotenuse, α and β angles opposite to the catheti); $\alpha + \beta = \pi/2$ rad.



sen $\alpha = a/c$; cos $\alpha = b/c$; tg $\alpha = a/b$; ctg $\alpha = b/a$

$a = c$ sen $\alpha = c$ cos $\beta = b$ tg α

$b = c$ cos $\alpha = c$ sen $\beta = a$ tg β

$a^2 + b^2 = c^2$; $c = \sqrt{a^2 + b^2}$ (Pythagoras theorem)

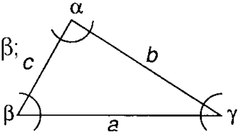
B) Oblique-angled triangle (*a, b, c* being the triangle sides; α, β, γ being their relevant opposite angles); $\alpha + \beta + \gamma = \pi$ rad $= 180$

$a/$sen $\alpha = b/$sen $\beta = c/$sen γ (theorem of sines)

$c^2 = a^2 + b^2 - 2\ a\ b \cos \gamma$ (Carnot theorem)

— From the two sides *a, b* and angle γ, find the third side *c* and angles α and β.
  $c = \sqrt{a^2 + b^2 - 2\ ab \cos \gamma}$; sen $\alpha = a$ sen γ/c; $\alpha = ...$;
  $\beta = 180 - \alpha - \gamma$.

— From the two sides *a, b* and angle α, find the third side *c* and angles β and γ.
  sen $\beta = b$ sen α/a; $\beta = ...$; $\gamma = 180 - \alpha - \beta$;
  $c = \sqrt{a^2 + b^2 - 2\ ab\ \cos \gamma}$.

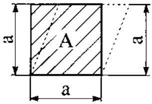


— Given the three sides, find the angles
  cos $\gamma = (a^2 + b^2 - c^2)\ /\ (2\ ab)$; $\gamma = ...$; sen $\alpha =$ sen γ/c;
  $\alpha = ...$; $\beta = 180 - \alpha - \gamma$.

— Given two angles α, β and a leg *a*, find the third angle γ and the other two sides *b, c*.
  $\gamma = 180 - \alpha - \beta$; $b = a$ sen β / sen α; $c = a$ sen γ / sen α

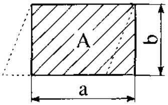
— Given a side *c* and two adjacent angles α, β, find the third angle γ and the other two sides.
  $\gamma = 180 - \alpha - \beta$; $b = c$ sen $\beta/$sen γ; $a = c$ sen $\alpha/$sen γ

CALCULATION OF AREAS, PERIMETER



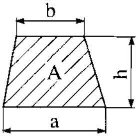
Square, Rhombus

$A = a^2$; $P = 4 \cdot a$



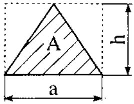
Rectangle, parallelogram

$A = a \cdot b$; $P = 2 \cdot (a + b)$; $a = \frac{P}{2} - b$



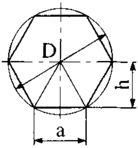
Trapezium

$A = \frac{a + b}{2} \cdot h$; $a = \frac{2A}{h} - b$



Triangle

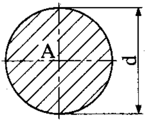
$A = \frac{a \cdot h}{2}$; $a = \frac{2 \cdot A}{h}$; $h = \frac{2 \cdot A}{a}$



Hexagon

$A = \frac{a \cdot h}{2} \cdot n = 3 \cdot a \cdot h$;

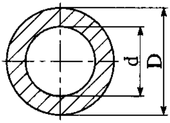
A = Area
P = Perimeter
n = Number of sides



Circle

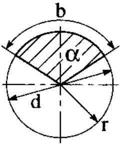
$$A = \frac{d^2 \cdot \pi}{4} = 0{,}7854 \cdot d^2;$$

$$P = d \cdot \pi \,;\ d = \sqrt{\frac{A}{0{,}7854}}$$



Circular ring

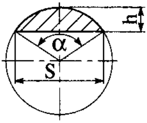
$$A = \frac{\pi}{4}\,(D^2 - d^2) = 0{,}7854\,(D^2 - d^2)$$



Circular sector

$$A = \frac{b \cdot r}{2} = 0{,}7854\,\frac{d^2 \cdot \alpha}{360^\circ} = \frac{\pi \cdot r^2 \cdot \alpha}{360^\circ}$$

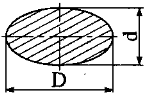
$$b = \frac{r \cdot \pi \cdot \alpha}{180}\,;\ b = \frac{\pi \cdot d \cdot \alpha}{360^\circ}\,;\ d = \frac{360^\circ \cdot b}{\pi \cdot \alpha}$$



Circular segment

$$A = \pi\,\frac{r^2 \cdot \alpha}{360^\circ} - \frac{s\,(r - h)}{2} \approx \frac{2}{3} \cdot s \cdot h$$

$$h = \frac{A \cdot 3}{s\,2} \quad S = 2\,\sqrt{h\,(2\,r - h)}$$

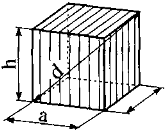


Ellipse

$$A = 0{,}7854\,D \cdot d = \frac{D \cdot d \cdot \pi}{4}\,;\ P \approx \frac{D + d}{2}$$

A = Area
P = Perimeter
d = Diameter; lower half-axis
D = Diameter; higher half-axis

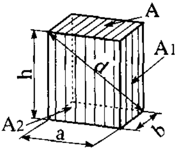
CALCULATION OF VOLUMES, LATERAL AREAS, TOTAL AREAS



Cube

$V = a^3$; $d = a \cdot \sqrt{3}$

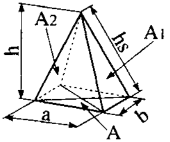
$a = \sqrt[3]{V}$; $At = 6 \cdot a^2$; $Al = 4 \cdot a^2$



Right prisme

$V = a \cdot b \cdot h = A \cdot h$; $At = 2\ (A + A_1 + A_2)$

$d = \sqrt{a^2 + h^2 + b^2)}$ $\quad Al = 2\ (A_1 + A_2)$

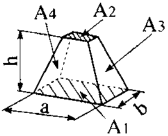


Pyramid

$$V = \frac{1}{3}\, a \cdot b \cdot h = \frac{A \cdot h}{3} ;$$

$At = A + 2\ (A_1 + A_2)$

$$hs = \sqrt{\left(\frac{a^2 + b^2}{4}\right) + h^2}$$



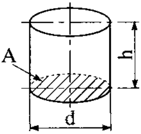
Frustum of pyramid

$$V = \frac{h}{3}\ (A_1 + A_2 + \sqrt{A_1 \cdot A_2}) = \frac{A_1 + A_2}{2} \cdot h$$

$At = A_1 + A_2 + 2\ (A_3 + A_4)$

$Al = 2 \cdot (A_3 + A_4)$

A_t = Total area
V = Volume
Al = Lateral area
h = Height
d = Diagonal

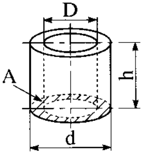


Cylinder

$$V = A \cdot h = \frac{d^2 \cdot \pi}{4} \cdot h = 0{,}7854 \cdot d^2 \cdot h$$

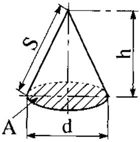
$$Al = \pi \cdot d \cdot h$$

$$At = 2\,A + d \cdot \pi \cdot h$$



Hollow cylinder

$$V = A \cdot h = 0{,}7854 \cdot (D^2 - d^2) \cdot h$$

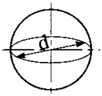


Straight cone

$$V = \frac{A \cdot h}{3} = \frac{d^2 \cdot 0{,}7854 \cdot h}{3};$$

$$Al = \pi \cdot r \cdot \sqrt{r^2 + h^2} = \pi \cdot r \cdot s$$

$$At = A + Al$$



Sphere

$$V = \frac{4}{3} \cdot \pi \cdot r^3 = \frac{d^3 \cdot \pi}{6} = 0{,}5236 \cdot d^3$$

$$At = \pi \cdot d^2; \quad d = \sqrt{\frac{6 \cdot V}{\pi}}$$

A = Base area

At = Total area

Al = Lateral area

INTERNATIONAL SYSTEM MEASURING UNITS

Basic units

IS basic units

Unit	Unit Symbol	Denomination
Length	m	metre
Mass	kg	kilogram
Time	s	second
Electrical current intensity	A	ampere
Thermodynamic temperature	K	Kelvin
Light intensity	cd	candle

Unit decimal multiples and sub multiples

Power of ten	Prefix	Symbol
10^{12}	earth	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

Derived units

***newton* (N)**: a force exerting an acceleration of 1 m/s^2 to a body with a mass of 1 kg;

***pascal* (Pa)**: pressure force 1 N exerted on a surface with area of 1 m^2. The bar unit is also used (1 bar $=10^5$ Pa);

***joule* (J)**: work done when the point of application of a force of 1 N is displaced through a distance of 1 m in the direction of the force;

***watt* (W)**: the power of a system producing the work of 1 J in 1 S;

***coulomb* (C)**: an electric charge that in 1 S passes through a conductor having a current flow of 1 A;

***volt* (V)**: potential difference between two sections of a conductor with 1 A current flow which dissipates 1 W of power between the sections;

***farad* (F)**: the capacity of a capacitor in which the transfer of 1 C from one armature to the other causes a potential difference of 1 V;

ohm (Ω): electrical resistance between two sections of a conductor having a potential difference of 1 V if the current is 1 A;

***weber* (Wb)**: magnetic induction flow (1 Wb =1 V.s);

***tesla* (T)**: magnetic induction (1 T =1 Wb/m^2);

***henry* (H)**: inductance (1 H =1 V.s/A).

Length conversion

A \ B	mm	cm	m	in	ft	yd	km	mile
mm	1	10^{-1}	10^{-3}	$3{,}93701 \cdot 10^{-2}$	$3{,}28084 \cdot 10^{-3}$	$1{,}09361 \cdot 10^{-3}$	10^{-6}	$6{,}21371 \cdot 10^{-7}$
cm	10	1	10^{-2}	$3{,}93701 \cdot 10^{-1}$	$3{,}28084 \cdot 10^{-2}$	$1{,}09361 \cdot 10^{-2}$	10^{-5}	$6{,}21371 \cdot 10^{-6}$
m	1000	100	1	39,3701	3,28084	1,09361	10^{-3}	$6{,}21371 \cdot 10^{-4}$
in	25,4	2,54	$2{,}54 \cdot 10^{-2}$	1	$8{,}33333 \cdot 10^{-2}$	$2{,}77778 \cdot 10^{-2}$	$2{,}54 \cdot 10^{-5}$	$1{,}57828 \cdot 10^{-5}$
ft	304,8	30,48	$3{,}048 \cdot 10^{-1}$	12	1	$3{,}33333 \cdot 10^{-1}$	$3{,}048 \cdot 10^{-4}$	$1{,}89394 \cdot 10^{-4}$
yd	914,4	91,44	$9{,}144 \cdot 10^{-1}$	36	3	1	$9{,}144 \cdot 10^{-4}$	$5{,}68182 \cdot 10^{-4}$
km	10^{6}	10^{5}	1000	39370,1	3280,84	1093,61	1	$6{,}21371 \cdot 10^{-1}$
mile	$1{,}60934 \cdot 10^{6}$	160934	1609,34	63360	5280	1760	1,60934	1

Surface conversion

A \ B	cm^2	m^2	ha	km^2	in^2	ft^2	yd^2	$mile^2$
cm^2	1	10^{-4}	10^{-8}	10^{-10}	$1{,}55000 \cdot 10^{-1}$	$1{,}07639 \cdot 10^{-3}$	$1{,}19599 \cdot 10^{-4}$	$3{,}86102 \cdot 10^{-11}$
m^2	10000	1	10^{-4}	10^{-6}	1550,00	10,7639	1,19599	$3{,}86102 \cdot 10^{-7}$
ha	10^{8}	10000	1	10^{-2}	$1{,}55000 \cdot 10^{7}$	107 639	11959,9	$3{,}86102 \cdot 10^{-3}$
km^2	10^{10}	10^{6}	100	1	$1{,}55000 \cdot 10^{9}$	$1{,}07639 \cdot 10^{7}$	$1{,}19599 \cdot 10^{6}$	$3{,}86102 \cdot 10^{-1}$
in^2	6,45160	$6{,}45160 \cdot 10^{-4}$	$6{,}45160 \cdot 10^{-8}$	$6{,}45160 \cdot 10^{-10}$	1	$6{,}94444 \cdot 10^{-3}$	$7{,}71605 \cdot 10^{-4}$	$2{,}49098 \cdot 10^{-10}$
ft^2	929,030	$9{,}29030 \cdot 10^{-2}$	$9{,}29030 \cdot 10^{-6}$	$9{,}29030 \cdot 10^{-8}$	144	1	$1{,}11111 \cdot 10^{-1}$	$3{,}58701 \cdot 10^{-8}$
yd^2	8361,27	$8{,}36127 \cdot 10^{-1}$	$8{,}36127 \cdot 10^{-5}$	$8{,}36127 \cdot 10^{-7}$	1296	9	1	$3{,}22831 \cdot 10^{-7}$
$mile^2$	$2{,}58999 \cdot 10^{10}$	$2{,}58999 \cdot 10^{6}$	258,999	2,58999	$4{,}01449 \cdot 10^{9}$	$2{,}78784 \cdot 10^{7}$	$3{,}09760 \cdot 10^{6}$	1

Volume conversion

A \ B	cm^3	$dm^3 = l$	in^3	ft^3	yd^3	US gal	Imp gal
cm^3	1	10^{-3}	$6{,}10237 \cdot 10^{-2}$	$3{,}53147 \cdot 10^{-5}$	$1{,}30795 \cdot 10^{-6}$	$2{,}64172 \cdot 10^{-4}$	$2{,}19969 \cdot 10^{-4}$
$dm^3 = l$	1000	1	61,0237	$3{,}53147 \cdot 10^{-2}$	$1{,}30795 \cdot 10^{-3}$	$2{,}64172 \cdot 10^{-1}$	$2{,}19969 \cdot 10^{-1}$
in^3	16,3871	$1{,}63871 \cdot 10^{-2}$	1	$5{,}78704 \cdot 10^{-4}$	$2{,}14335 \cdot 10^{-5}$	$4{,}32900 \cdot 10^{-3}$	$3{,}60465 \cdot 10^{-3}$
ft^3	28316,8	28,3168	1728	1	$3{,}70370 \cdot 10^{-2}$	7,48052	6,22884
yd^3	764 555	764,555	46656	27	1	201,974	168,179
US gal	3785,41	3,78541	231	$1{,}33681 \cdot 10^{-1}$	$4{,}95113 \cdot 10^{-3}$	1	$8{,}32674 \cdot 10^{-1}$
Imp gal	4546,09	4,54609	277,419	$1{,}60544 \cdot 10^{-1}$	$5{,}94606 \cdot 10^{-3}$	1,20095	1

Mass conversion

A \ B	g	kg	oz	lbm
g	1	10^{-3}	$3,52740 \cdot 10^{-2}$	$2,20462 \cdot 10^{-3}$
kg	1000	1	35,2740	2,20462
oz	28,3495	$2,83495 \cdot 10^{-2}$	1	$6,25 \cdot 10^{-2}$
lbm	453,592	$4,53592 \cdot 10^{-1}$	16	1

Energy conversion

A \ B	J	Wh	kp m	kcal
J	1	$2,77778 \cdot 10^{-4}$	$1,01972 \cdot 10^{-1}$	$2,38846 \cdot 10^{-4}$
Wh	3600	1	367,098	$8,59845 \cdot 10^{-1}$
kp m	9,80665	$2,72407 \cdot 10^{-3}$	1	$2,34228 \cdot 10^{-3}$
kcal	4186,8	1,163	426,935	1

Torque conversion

A \ B	cm N	m N	cm kp	m kp	cm grp	in lbs	ft lbs
cm N	1	10^{-2}	$1,01972 \cdot 10^{-1}$	$1,01972 \cdot 10^{-3}$	101,972	$8,85075 \cdot 10^{-2}$	$7,37562 \cdot 10^{-3}$
m N	100	1	10,1972	$1,01972 \cdot 10^{-1}$	10197,2	8,85075	$7,37562 \cdot 10^{-1}$
cm kp	9,80665	$9,80665 \cdot 10^{-2}$	1	10^{-2}	1000	$8,67962 \cdot 10^{-1}$	$7,23301 \cdot 10^{-2}$
m kp	980,665	9,80665	100	1	10^{5}	86,7962	7,23301
cm grp	$9,80665 \cdot 10^{-3}$	$9,80665 \cdot 10^{-5}$	10^{-3}	10^{-5}	1	$8,67962 \cdot 10^{-4}$	$7,23301 \cdot 10^{-5}$
in lbs	11,2985	$1,12985 \cdot 10^{-1}$	1,15212	$1,15212 \cdot 10^{-2}$	1152,12	1	$8,33333 \cdot 10^{-2}$
ft lbs	135,582	1,35582	13,8225	$1,38255 \cdot 10^{-1}$	13825,5	12	1

Inertia conversion

A \ B	kg cm²	kp cm s²	kg m²	kp m s²	Lb in²	Lb in s²	Lb ft²	Lb ft s²
kg cm²	1	$1{,}01972 \cdot 10^{-3}$	10^{-4}	$1{,}01972 \cdot 10^{-5}$	$3{,}41717 \cdot 10^{-1}$	$8{,}85075 \cdot 10^{-4}$	$2{,}37304 \cdot 10^{-3}$	$7{,}37562 \cdot 10^{-5}$
kp cm s²	980,665	1	$9{,}80665 \cdot 10^{-2}$	10^{-2}	335,110	$8{,}67962 \cdot 10^{-1}$	2,32715	$7{,}23301 \cdot 10^{-2}$
kg m²	10^{4}	10.1972	1	$1{,}01972 \cdot 10^{-1}$	3417,17	8,85075	23,7304	$7{,}37562 \cdot 10^{-1}$
kp m s²	98066,5	100	9,80665	1	33511,0	86,7962	232,715	7,23301
Lb in²	2,92640	$2{,}98409 \cdot 10^{-3}$	$2{,}92640 \cdot 10^{-4}$	$2{,}98409 \cdot 10^{-5}$	1	$2{,}59008 \cdot 10^{-3}$	$6{,}94444 \cdot 10^{-3}$	$2{,}15840 \cdot 10^{-4}$
Lb in s²	1129,85	1,15212	$1{,}12985 \cdot 10^{-1}$	$1{,}15212 \cdot 10^{-2}$	386,089	1	2,68117	$8{,}33333 \cdot 10^{-2}$
Lb ft²	421,401	$4{,}29710 \cdot 10^{-1}$	$4{,}21401 \cdot 10^{-2}$	$4{,}29710 \cdot 10^{-3}$	144	$3{,}72971 \cdot 10^{-1}$	1	$3{,}10810 \cdot 10^{-2}$
Lb ft s²	13558,2	13,8255	1,35582	$1{,}38255 \cdot 10^{-1}$	4633,06	12	32,1740	1

Force conversion

A \ B	N	kp	grp	lbf
N	1	$1,01972 \cdot 10^{-1}$	101,972	$2,24809 \cdot 10^{-1}$
kp	9,80665	1	1000	2,20462
grp	$9,80665 \cdot 10^{-3}$	10^{-3}	1	$2,20462 \cdot 10^{-3}$
lbf	4,44822	$4,53592 \cdot 10^{-1}$	453,592	1

Power conversion

A \ B	kW	PS	HP	kpm/s	kcal/s
kW	1	1,35962	1,34102	101,972	$2,38846 \cdot 10^{-1}$
PS	$7,35499 \cdot 10^{-1}$	1	$9,86320 \cdot 10^{-1}$	75	$1,75671 \cdot 10^{-1}$
HP	$7,45700 \cdot 10^{-1}$	1,01387	1	76,0402	$1,78107 \cdot 10^{-1}$
kp m/s	$9,80665 \cdot 10^{-3}$	$1,33333 \cdot 10^{-2}$	$1,31509 \cdot 10^{-2}$	1	$2,34228 \cdot 10^{-3}$
kcal/s	4,1868	5,69246	5,61459	426,935	1

SYMBOLS AND MEASURING UNITS ACCORDING TO THE INTERNA TIONAL SYSTEM, USED IN POWER TRANSMISSION TECHNOLOGY

Symbol	Meaning	IS unit symbol
Geometry		
A	Area	m^2
a	Distance	m
α, β, γ ...	Angle	rad
b	Width	m
d	Thickness	m
d	Diameter	m
h	Height	m
l	Length	m
r	Radius	m
s	Space	m
V	Volume	m^3
Time		
a	Acceleration	m/s^2
α	Angular acceleration	rad/s^2
f	Frequency	Hz
g	Gravity Acceleration	m/s^2
n	Rotation speed	1/s
ω	Angular speed	rad/s
T	Time constant	s
t	Time, duration	s
v	Speed	m/s

Symbol	Meaning	IS unit symbol
Mechanics		
E	Young's elasticity	MPa
F	Force	N
G	Weight	N
J	Moment of inertia	kgm^2
M	Torque	Nm
m	Mass	kg
P	Power	W
P	Pressure	Pa
Q	Density	kg/m^3
σ,	Tensile, compressive and bending stress	Pa
W	Work, energy	J
η	Performance	—
μ	Coefficient of friction	—

BASIC FORMULAS USED IN POWER TRANSMISSION TECHNOLOGY

Translation		Rotation	
$s = v \cdot t$	Space (m)	angle	$\varphi = \omega t = 2\pi \cdot n \cdot t$
$v = \frac{s}{t}$	Linear speed (m/s)		$v = d\pi n = \omega r$
	Angular speed (rad/s)		$\omega = \dot{\varphi} = 2\pi n = \frac{v}{r}$
$a = \frac{v}{t}$	Acceleration (m/s²)		$\dot{\omega} = \ddot{\varphi} = \frac{\omega}{t}$
$F = m \cdot a$	Force (N)		$F = m\, r\, \dot{\omega}$
$M = F \cdot r$	Torque (Nm)		$M = J \cdot \dot{\omega}$
$P = F \cdot v$	Power (Watt)		$P = M \cdot \omega$
$W = F \cdot s$	Energy (Joule)		$W = M \cdot \varphi$
$W = \frac{1}{2} mv^2$	Energy (Joule)		$W = \frac{1}{2} J \omega^2$

Important definitions

1 Newton (N) = 1 kgm/s²	Force
1 Kilogram-weight (kp) = 9,80665 N	Force
1 metric horsepower PS = 735,5 W = 75 kgm/s	Power
1 horsepower (HP) = 745,7 W	Power
1 Wh/3600 = 1 Nms = 1 Joule (J)	Work, energy
g = 9,80665 m/s²	Gravity acceleration

SYMBOLS AND DESCRIPTIONS

M = peak or motor total torque (Nm)

M_L = stall torque (Nm)

M_a = acceleration torque (Nm)

M_{fr} = braking torque (Nm)

P = motor total power (kW)

P_L = power at normal operating speed (kW)

P_a = acceleration power (kW)

n = rotating speed (min^{-1})

Δn = rotation difference (min^{-1})

v = linear speed (m/min)

Δv = speed difference (m/min)

J = inertia (kgm^2)

m = mass (kg)

F = force (N)

W = energy (J)

t_a = acceleration time (s)

t_{fr} = braking time (s)

s = space (m)

d = diameter (mm)

r = radius (mm)

μ = coefficient of friction

p = pressure (N/m^2 or Pa)

g = 9,80665 m/s^2

π = 3,141592654

Linear speed (m/min)

$$v = \frac{d \cdot \pi n}{1000}$$

Force (N)

$$F = 1000 \frac{M}{r} = \mu \cdot m \cdot g$$

Torque (Nm)

$$M = \frac{F \cdot r}{1000}$$

$$M = \frac{3 \cdot 10^4 P}{\pi \cdot n} = \frac{9549 P}{n}$$

Work (Joule)

$$W = F \cdot s = m \cdot g \cdot s$$

Translation energy (Joule)

$$W = \frac{m v^2}{7200}$$

Rotation energy (Joule)

$$W = \frac{\pi^2}{1800} J n^2 = \frac{J n^2}{182{,}4}$$

Power (kW)

in rotating conditions

$$P = \frac{\pi}{30} \cdot 10^{-3} M \cdot n = \frac{M \cdot n}{9549}$$

in translating conditions

$$P = \frac{F \cdot v}{6 \cdot 10^4}$$

in lifting conditions

$$P = \frac{m \cdot g \cdot v}{6 \cdot 10^4}$$

Important definitions

$$\eta = \frac{P_{available}}{P_{absorbed}} \quad \text{Efficiency}$$

$$u = \frac{n_1}{n_2} = \frac{M_2}{M_1} = \sqrt{\frac{J_2}{J_1}} \quad \text{Ratio}$$

Transmission acceleration

Total torque (Nm)

$$M = M_L + M_a = M_L + \frac{\pi}{30} J \frac{\Delta n}{t_a}$$

Acceleration torque (Nm)

$$M_a = \frac{\pi}{30} J \frac{\Delta n}{t_a} = 0{,}105 J \frac{\Delta n}{t_a}$$

knowing that

$$n = \frac{1000 v}{d \cdot \pi}$$

$$M_a = \frac{100}{3d} J \frac{\Delta v}{t_a}$$

effective work (Joule)

$$W = \frac{\pi^2}{1800} J \Delta n^2 \frac{M}{M - M_L} = \frac{J \Delta n^2 M}{182{,}4 (M - M_L)}$$

$$W = \frac{5000}{9} J \frac{\Delta v^2}{d^2} \frac{M}{M - M_L}$$

Total power (kW)

$$P = P_L + P_a$$

Power under normal operating conditions (kW)

$$P_L = \frac{\pi \cdot n \cdot M_L}{3 \cdot 10^4} = \frac{n \cdot M_L}{9549} = \frac{v \cdot M_L}{30 \cdot d}$$

Power when accelerating (kW)

$$P_a = \frac{\pi^2 n}{9 \cdot 10^5} J \frac{\Delta n}{t_a} = \frac{n J \Delta n}{9{,}12 \cdot 10^4 \cdot t_a}$$

$$P_a = \frac{10 v}{9 d^2} J \frac{\Delta v}{t_a} = \frac{m \cdot v \cdot \Delta v}{7{,}2 \cdot 10^6 t_a}$$

For braking, symbols Δ and M_a should be modified.

Acceleration time

$$t_a = \frac{\pi}{30} J \frac{\Delta n}{M - M_L} = 0{,}105 \frac{J \Delta n}{M - M_L} = \frac{100 J}{3d} \frac{\Delta v}{M - M_L}$$

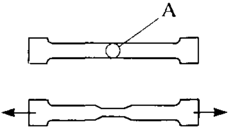
$$t_a = \frac{\pi^2 n J \Delta n}{9 \cdot 10^5 (P - P_L)} = \frac{n J \Delta n}{9{,}12 \cdot 10^4 (P - P_L)}$$

$$t_a = \frac{J \cdot \Delta n}{9{,}55 \cdot M_a} ; t_{fr} = \frac{J \cdot \Delta n}{9{,}55 \quad M_{fr}}$$

Acceleration during horizontal movement

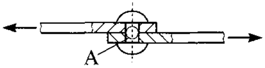
$$P = \frac{m v}{6 \cdot 10^4} \left(\mu \cdot g + \frac{\Delta v}{60 t_a} \right)$$

STRENGTH OF MATERIALS



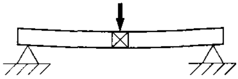
Tensile strength

$$\sigma = \frac{F}{A} \qquad F = \sigma \cdot A$$



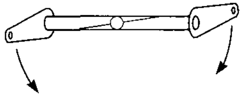
Shear strength

$$\tau = \frac{F}{A} \qquad F = A \cdot \tau$$



Bending strength

$$\sigma = \frac{M}{W_b} \qquad [N/mm^2]$$

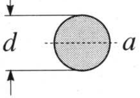
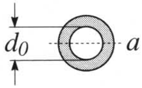
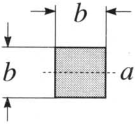
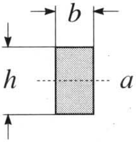


Torsional strength

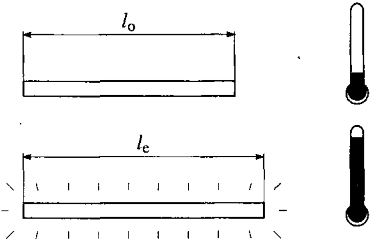
$$\tau = \frac{M}{W_t} \qquad [N/mm^2]$$

A = Area of section in mm^2
σ = Torsional and bending strength in N/mm^2
τ = Shear and torsional strength in N/mm^2
F = Force in N
M = Moment in Nmm
W_b = Bending strength module in mm^3
W_t = Torsional strength module in mm^3

Moment of inertia - Resistance module

	Resistance module	Moment of inertia of a surface
	$W_b = \frac{\pi}{32} \cdot d^3$	$I_a = \frac{\pi}{64} \cdot d^4$
	$W_t = \frac{\pi}{16} \cdot d^3$	$I_p = \frac{\pi}{32} \cdot d^4$
	$W_b = \frac{\pi}{32} \cdot (d^4 - d_0^4)/d$	$I_a = \frac{\pi}{64} \cdot (d^4 - d_0^4)$
	$W_t = \frac{\pi}{16} \cdot (d^4 - d_0^4)/d$	$I_a = \frac{\pi}{32} \cdot (d^4 - d_0^4)$
	$W_b = \frac{b^3}{6}$	$I_a = \frac{b^4}{12}$
	$W_t = \frac{2}{9} \cdot b^3$	
	$W_b = \frac{1}{6} \cdot b \cdot h^2$	
	$W_t = \frac{2}{9} \cdot b^2 \cdot h$	$I_a = \frac{h^3 b}{12}$

THERMAL EXPANSION - ELONGATION



Elongation

$$l_v = \alpha \cdot l_o \,(t_2 - t_1)$$

Final length

$$l_f = l_o \,(1 + \alpha \cdot \Delta T)$$

$$l_o = \frac{l_v}{\alpha \cdot \Delta T}$$

$$\Delta T = \frac{l_v}{\alpha \cdot l_o}$$

l_v = Elongation
l_o = Initial length
l_f = Final length (after heating)
ΔT = Temperature difference in Kelvin
α = Thermal expansion coefficient for 1 degree

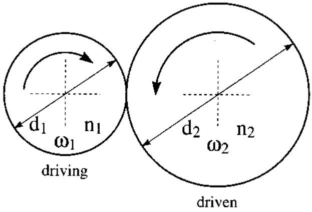
Thermal expansion coefficient for 1K and length unit (between 0 and 100°C)

Aluminium	0,000024
Bronze	0,000018
Glass	0,000009
Cast-iron	0,000011
Copper	0,000017
Magnesium	0,000025
Brass	0,000019
Steel	0,000012

GEAR WHEELS

The ratio between a driving wheel with diameter d_1 and a driven wheel with diameter d_2 is defined as **transmission ratio** and is indicated as u.

$$u = \frac{d_2}{d_1} = \frac{n_1}{n_2} = \frac{\omega_1}{\omega_2}$$



In gear wheels

$$u = \frac{z_2}{z_1}$$

as:

n_1 = angular speed, in $\frac{revs}{min}$ of the driving wheel

n_2 = angular speed, in $\frac{revs}{min}$ of the driven wheel

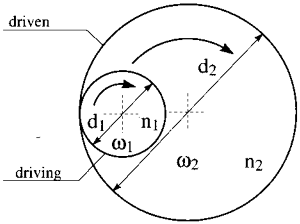
ω_1 = angular speed, in $\frac{rad}{s}$ of the driving wheel

ω_2 = angular speed, in $\frac{rad}{s}$ of the driven wheel

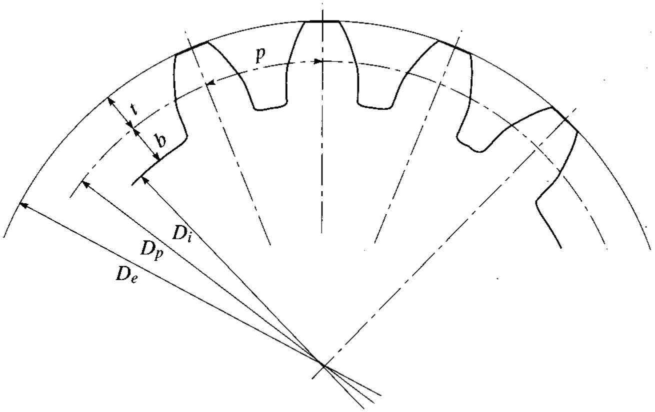
z_1 = number of teeth of the driving wheel

z_2 = number of teeth of the driven wheel

When $u > 1$, the gear reduces speed, when < 1, the gear multiplies speed.
When drive is transmitted between external gear-wheels, rotation directions are opposite. When one of the two gear-wheels is internal, rotation directions are identical.



Elements of a cylindrical gear wheel with helical spur teeth and an involute-to-circle profile



z = number of teeth on gear wheel
t = tooth addendum in mm
m = module in mm
b = tooth dedendum is $\frac{7}{6}$ m in mm
D_e = external diameter in mm
D_p = pitch diameter in mm
D_i = internal diameter in mm
p = pitch in mm
α = pressure angle

Relations between elements of a helical spur teeth cylindrical gear wheel

$m = \frac{D_p}{z}$ [mm] from which

$D_p = m \cdot z$; $z = \frac{D_p}{m}$

$p = \frac{\pi D_p}{z}$ [mm] from which

$\frac{p}{\pi} = \frac{D_p}{z} = m$ [mm]

$p = \pi m$ [mm]

Forces transmitted by a helical spur teeth cylindrical gear wheel set

The **tangential force** T is the component of force F acting in the direction of the tangent common to the two pitch circumferences - the gear wheel rotates by action of T.
The **radial force** R is the component of force F directed towards the gear wheel centre and is normal to the axis of the gear wheel.

$T = \frac{9550\,P}{r\,n}$ [N]; $R = T\,tg\alpha$ [N]; $F = \frac{T}{\cos\alpha}$ [N]

where r = pitch radius [m]
P = power [kW]
n = rpm [min^{-1}]

$M = \frac{9550\,P}{n}$ [Nm] transmitted torque

Main relations between the elements of a cylindrical gear wheel with helical teeth

z = number of teeth
p_c = circumference pitch
p_n = normal pitch
p_a = axial pitch
p_e = helix pitch
m_c = circumference module
m_n = normal module
m_a = axial module
α = pressure angle
β = helix angle

$D_p = m_c z$

$p_n = p_c \cos\beta$

$$p_c = \frac{p_n}{\cos\beta}$$

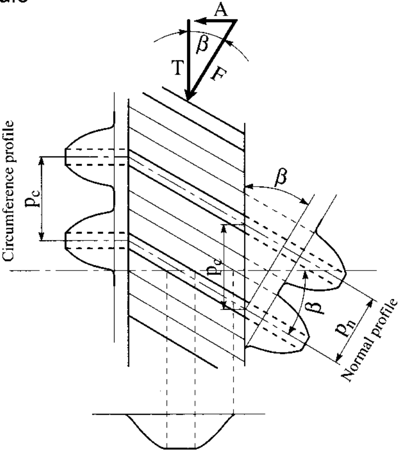
$p_n = \pi m_n$

$p_c = \pi m_c$

$$\frac{\pi Dp}{p_e} = \operatorname{tg}\beta \text{ from which}$$

$$p_e = \frac{\pi D_p}{\operatorname{tg}\beta}$$

$$p_a = \frac{p_e}{z}$$



Loads transmitted between cylindrical helical teeth gear wheels with parallel axis

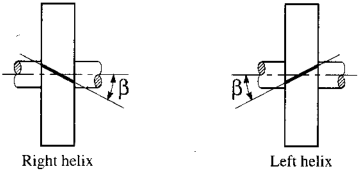
$$T = \frac{9550\,P}{r\,n} \qquad A = T \operatorname{tg}\beta$$

where r = pitch radius [m]
P = power [kW]
n = rpm [min^{-1}]

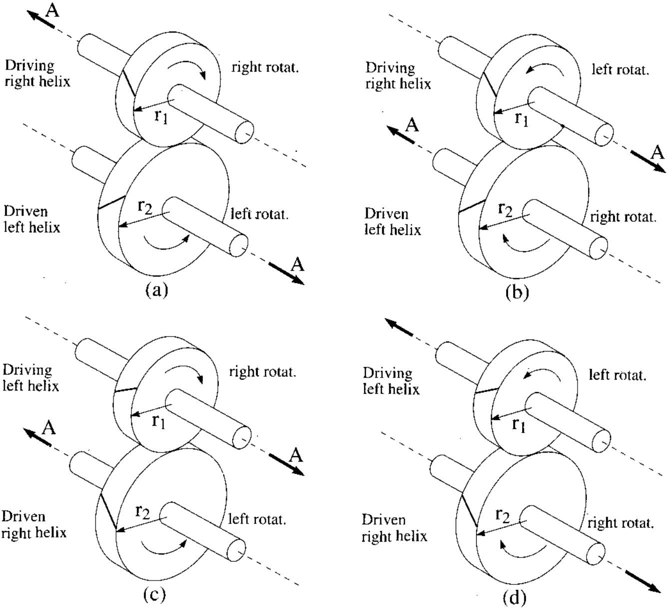
$$F = \frac{T}{\cos\beta} \qquad R = \frac{T \operatorname{tg}\alpha}{\cos\beta}$$

HELIX ANGLE DIRECTION

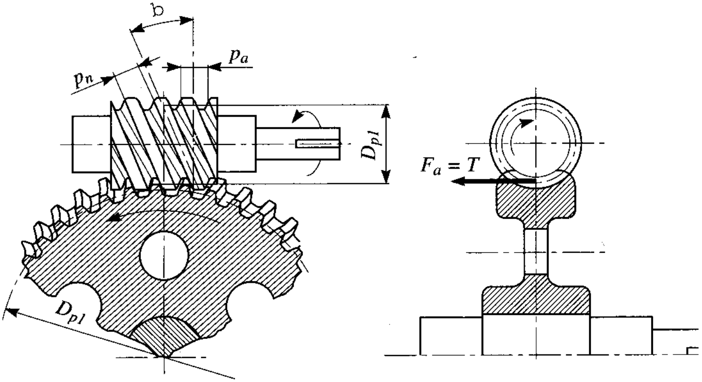
A helical teeth gear-wheel has its helix on the right if, where observing its profile, with the axis horizontally located, the teeth are inclined downwards to the right. It has its helix on the left if the teeth are inclined downwards to the left.



The direction of force A depends on the rotation direction of the two gear-wheels and on the direction of helix angle according to the following scheme:



HELICAL GEAR WHEEL WORM GEARING



p_n	=	normal pitch of worm and gear-wheel in mm
p_a	=	axial pitch of worm equal to circumference pitch of the gear-wheel in mm
p_e	=	worm helix pitch in mm
m_n	=	normal module in mm
m_{av}	=	axial module of worm equal to the circumference module of the gear-wheel, in mm
β	=	helix angle, worm and gear wheel
D_{p1}	=	worm pitch diameter in mm
D_{p2}	=	gear wheel pitch diameter in mm
i	=	number of worm starts
α	=	pressure angle
z	=	number of gear wheel teeth

Relations between the elements of a helical gear wheel with worm gearing

$p_n = \pi\, m_n$

$$p_a = \frac{\pi\, m_n}{\cos\beta} = \frac{p_n}{\cos\beta}\ ;\ p_e = \frac{p_n\, i}{\cos\beta}\ ;\ d_1 = \frac{m_n\, i}{\operatorname{sen}\beta}\ ;\ d_2 = \frac{m_n\, z}{\cos\beta}$$

Ratio

$$u = \frac{z}{i}$$

In the case of a worm with one start only $i = 1$ and $u = \frac{z}{1}$

Forces transmitted between worm and helical gear wheel

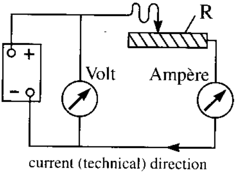
Tangent force of the worm fitted on the pitch circumference equal to gear-wheel axial force.

$$T = \frac{9550\, P}{r\, n}$$ = Gear wheel axial force, in N = worm tangent force

where r = worm pitch radius [m]
and P = power [kW]
n = rpm [min^{-1}]

$$R = \frac{T\, tg\, \alpha}{tg\, \beta}$$ = Gear wheel radial force = Worm radial force

$$A = \frac{T}{tg\, \beta}$$ = Gear wheel tangent force = Worm axial force



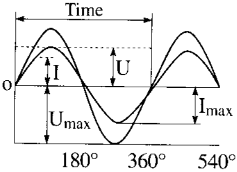
Ohm's law

Direct current

Voltage $U = R \cdot I$ [V]

Current $I = \frac{U}{R}$ [A]

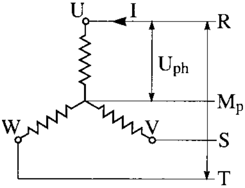
Resistance $R = \frac{U}{I}$ [Ω]



Alternated current

Voltage $U = 0{,}707 \cdot U_{max}$ [V]

Current $I = 0{,}707 \cdot I_{max}$ [A]

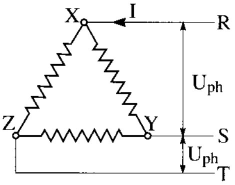


Three-phase current with star connection

Voltage $U = 1{,}73 \cdot U_{ph}$ [V]

ou $U = U_{ph} \sqrt{3}$

Current $I = I_{ph}$ [A]



Three-phase current with delta connection

Voltage $U = U_{ph}$ [V]

Current $I = 1{,}73 \cdot I_{ph}$ [A]

or $I = I_{ph} \cdot \sqrt{3}$

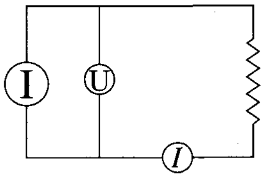
I_{ph} = Phase current in A

U_{ph} = Phase voltage in V

WORK AND ELECTRIC POWER

Direct current

Work $W = P \cdot t = U \cdot I \cdot t = [Ws]$



$$P = \frac{W}{t}$$

$$I = \frac{W}{U \cdot t}$$

$$t = \frac{W}{U \cdot I}$$

Power $P = U \cdot I [W]$

or

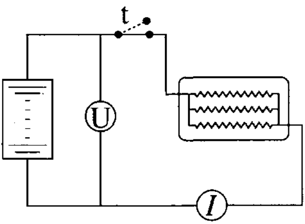
$P = I^2 \cdot R [W]$

or

$$P = \frac{U^2}{R} [W]$$

$$I = \sqrt{\frac{P}{R}} [A]$$

$U = \sqrt{P \cdot R} [V]$



Three-phase current

$P = U \cdot I \cdot 1{,}73 \cdot \cos \varphi = [W]$

P = Electric power in watt or kW

t = Time in seconds

W = Electrical work in watt · s

I = Current intensity in A

CHARACTERISTICS OF THE THREE-PHASE MOTOR

Absorbed power $P_{abs} = \frac{\sqrt{3} \cdot U \cdot I \cdot \cos\varphi}{1000}$

Available power $P_{del} = \frac{\sqrt{3} \cdot U \cdot I \cdot \cos\varphi \cdot \eta}{1000}$

P = power in kW
U = voltage in V
I = line current for phase in A
$\cos\varphi$ = power factor
η = motor performance

SYNCHRONOUS SPEED OF A THREE-PHASE ELECTRIC MOTOR

$$n_0 = 60 \frac{f}{p} = 120 \frac{f}{2p}$$

$$n = n_0 (1 - s) = 60 \frac{f}{p} (1 - s)$$

$$s = \frac{n_0 - n}{n_0}$$

n_0 = synchronous speed in min^{-1}
n = work speed in min^{-1}
f = main frequency in Hz
p = number of pole pairs
2p = number of poles
s = slipping

2p	f = 50 Hz	f = 60Hz	f = 100Hz	f = 200Hz	f = 400Hz	p
2	3000	3600	6000	12000	24000	1
4	1500	1800	3000	6000	12000	2
6	1000	1200	2000	4000	8000	3
8	750	900	1500	3000	6000	4
10	600	720	1200	2400	4800	5
12	500	600	1000	2000	4000	6

RELATION BETWEEN MOTOR SIZE AND POWER (CENELEC 231 - IEC 72)

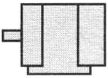
An example of the correlation between rated power at 4 poles and motor size.

Size	Rated power in kW
Axis height in mm	Closed motors with squirrel cage rotor
63	0,12
63	0,18
71	0,25
71	0,37
80	0,55
80	0,75
90 S	1,1
90 L	1,5
100 L	2,2
100 L	3
112 M	4
132 S	5,5
132 M	7,5
160 M	11
160 L	15
180 M	18,5
180 L	22
200 L	30

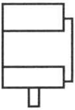
COMMON MOUNTING POSITIONS

The following table shows the most common mounting positions with reference to IEC 34-7 Standard.

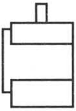
IM B 3
IM 1001



IM V 5
IM 1011



IM V 6
IM 1031



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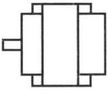

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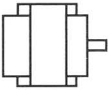

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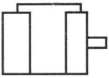
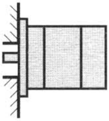
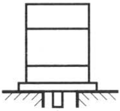


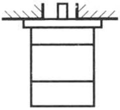
IM B 5
IM 3001



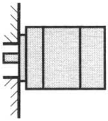
IM V 1
IM 3011



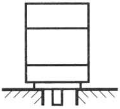
IM V 3
IM 3031



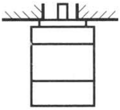
IM B 14
IM 3601



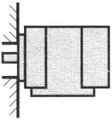
IM V 18
IM 3611



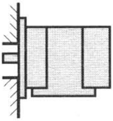
IM V 19
IM 3631



IM B 34
IM 2101



IM B 35
IM 2001



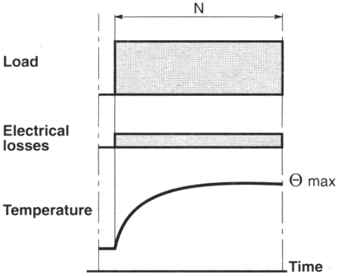
TYPES OF DUTY

Definitions

For a correct selection of the motor, customers must specify the foreseen types of duty.
Standards IEC 34-1 define 9 different types of duty from S1 to S9.

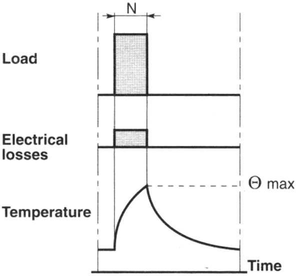
Continuous running duty - duty type S1

Operation at constant load of sufficient duration to reach thermal equilibrium



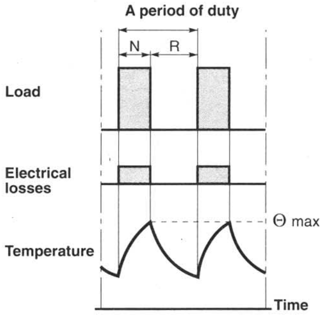
Short-time duty - duty type S2

Operation at constant load during a given time less than required to reach thermal equilibrium, followed by a rest enabling the machine to reach a temperature similar to that of the coolant (tolerance 2K).



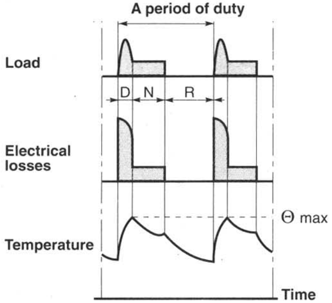
Intermittent periodic duty - duty type S3

A sequence of identical duty cycles, each including a period of operation at constant load and a rest (without connection to the mains). For this type of duty, the starting current does not significantly affect the temperature rise.



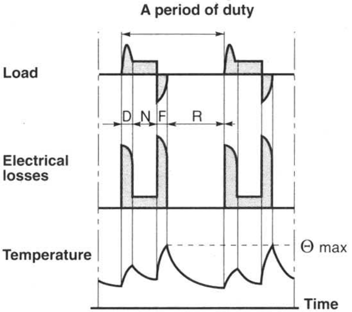
Intermittent periodic duty with starting - duty type S4

A sequence of identical duty cycles, each consisting of a significant period of starting, a period under constant load and a rest period.



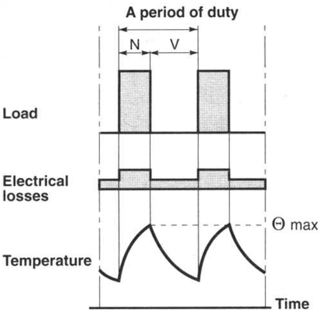
Intermittent periodic duty with electric braking - duty type S5

A sequence of identical duty cycles, each consisting of a period of starting, a period of operation at constant load followed by rapid electric braking, and a rest period.



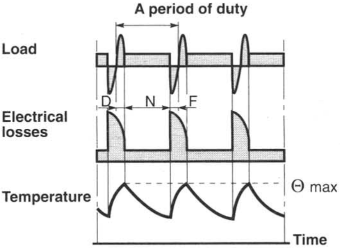
Continuous-operation periodic duty - duty type S6

A sequence of identical duty cycles, each consisting of a period of operation at constant load and a period of operation at no-load. There are no rest periods.



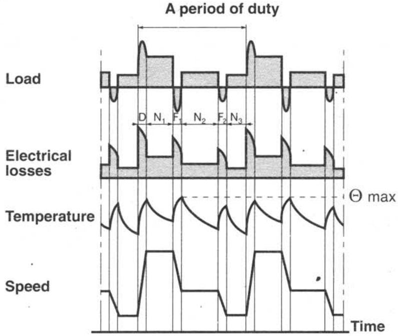
Continuous operation duty with electric braking - duty type S7

A sequence of identical duty cycles, each consisting of a period starting, a period of operation at constant load followed by electric braking. There are no rest periods.



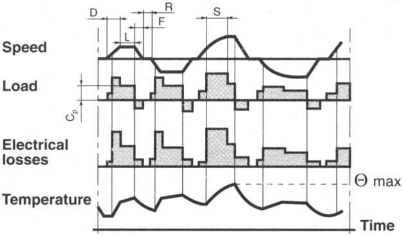
Continuous-operation periodic duty with related load/speed changes - duty type S8

A sequence of identical duty cycles, each consisting of a period of operation at constant load corrisponding to a predetermined speed of rotation, follewed by one or more periods of operation at other constant loads corresponding to different speeds of rotation,(e.g. duty with a switching pole induction motor). There are not rest periods. The period of duty is too short to reach thermal equilibrium.

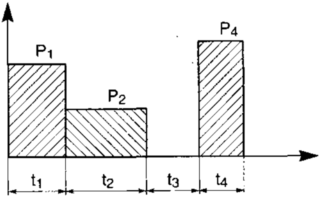


Duty with non-periodic load and speed variations - duty type S9

Duty in which generally load and speed vary non-periodically within the permissible range. This duty includes frequently overloads applied that may greatly exceed the full loads.



Power of equal thermal value both for intermittent duty and variable load.



$$P_t = \sqrt{\frac{P_1^2 \cdot t_1 + P_2^2 \cdot t_2 + P_4^2 t_4}{t_1 + t_2 + t_4 + t_3/4}}$$

$P = [W] =$ power

$t = [s] =$ time

How starting time is established

$$t = \frac{(J_M + J_L) \cdot \omega}{M}$$

(where $M = M_{Mm} - M_{Rm}$)

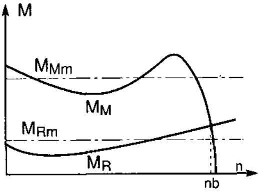
$J_M = [kg \cdot m^2] =$ moment of inertia of motor

$J_L = [kg \cdot m^2] =$ moment of inertia of load

$\omega = [RAD/S] =$ angular speed

$M_{Mm} = [N_m] =$ motor mean torque value

$M_{Rm} = [N_m] =$ mean resisting torque



Acoustic pressure level

$$L_{PA} = 20 \cdot \lg \left(\frac{p}{p_o}\right) [db]$$

$p = [N/m^2] =$ acustic pressure

where $p, p_o =$ acustic pressure

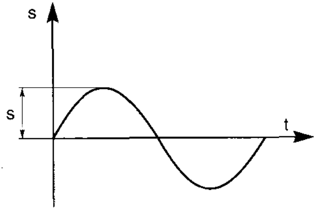
$p_o = 2 \cdot 10^{-5} N/m$

Acoustic power level

$$L_{WA} = L_p + 10 \cdot \lg \left(\frac{s}{s_o}\right) [db]$$

where $s =$ effective measuring area [m^2]
$s_o = 1 m^2 =$ reference area

vibration amplitude



$$s = \frac{\sqrt{2} \cdot V_{eff}}{2 \pi f} \; [mm]$$

where V_{eff} = vibration speed = [m/s]
f = vibration frequency = [s^{-1}]