An Iterative Uncoupling Technique for the Identification of the Dynamic Properties of Joints

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Abstract

The identification of the dynamic properties of joints is a very important subject in structural dynamics. However, classic uncoupling methods are often difficult to apply. Here, an iterative technique is proposed, trying to circumvent the ill-conditioning problems that arise due to any small perturbations existing in the experimental data. Various cases are discussed to illustrate the technique, including situations where it is not possible to take measurements on the joint itself; the results are compared with the ones obtained with other methods.

Theoretical formulation

To characterize the dynamic behavior of a joint, one uses the dynamic stiffnesses *Z* and the FRFs *H* of the various substructures, including the one representing the joint. Our complex structure *C* is formed by the coupling of two substructures, *A* and *B* (the joint) (Fig. 1). Substructure A is assumed as easy to model numerically; the internal co-ordinates are designated by *i* and those common to *B* are *j*, which are often difficult to measure. No internal co-ordinates are considered in *B*.

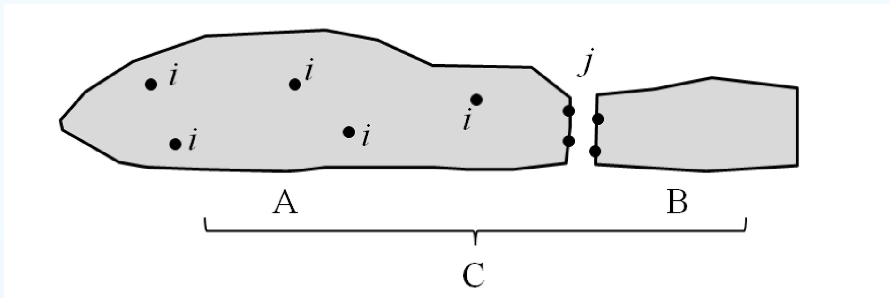

Figure 1 : Coupling of substructures *A* and *B*, to form structure *C*

Let us determine the dynamic stiffness of substructure *B*, Z^B, using the information obtained numerically from substructure *A* and the measurements taken at co-ordinates *i* of structure *C*. The dynamic stiffness Z^C may be written as:

$$\boldsymbol{Z}^C = \boldsymbol{Z}^A + \boldsymbol{Z}^B \tag{1}$$

Rewriting the r.h.s. of eq. (1) and inverting both sides, yields

$$\boldsymbol{H}^C = \left(\boldsymbol{I} + \boldsymbol{H}^A\boldsymbol{Z}^B\right)^{-1}\boldsymbol{H}^A \tag{2}$$

Explicitly writing (2) in terms of submatrices involving co-ordinates *i* and *j*, leads to

$$\begin{bmatrix} \boldsymbol{H}_{ii}^C & \boldsymbol{H}_{ij}^C \\ \boldsymbol{H}_{ji}^C & \boldsymbol{H}_{jj}^C \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I}_{ii} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{jj} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{ii}^A & \boldsymbol{H}_{ij}^A \\ \boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A \end{bmatrix}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{jj}^B \end{bmatrix}\right]^{-1}\begin{bmatrix} \boldsymbol{H}_{ii}^A & \boldsymbol{H}_{ij}^A \\ \boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A \end{bmatrix} \tag{3}$$

which, after various manipulations, gives

$$\begin{bmatrix} \boldsymbol{H}_{ii}^A & \boldsymbol{H}_{ij}^A \\ \boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}_{ii}^C & \boldsymbol{H}_{ij}^C \\ \boldsymbol{H}_{ji}^C & \boldsymbol{H}_{jj}^C \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ij}^A\boldsymbol{Z}_{jj}^B\left(\boldsymbol{I}_{jj}+\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\right)^{-1}\boldsymbol{H}_{ji}^A & \boldsymbol{H}_{ij}^A\boldsymbol{Z}_{jj}^B\left(\boldsymbol{I}_{jj}+\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\right)^{-1}\boldsymbol{H}_{jj}^A \\ \boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\left(\boldsymbol{I}_{jj}+\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\right)^{-1}\boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\left(\boldsymbol{I}_{jj}+\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\right)^{-1}\boldsymbol{H}_{jj}^A \end{bmatrix} \tag{4}$$

Pre-multiplying <u>the first element</u> of the first row of eq. (4) by $\left(\boldsymbol{H}_{ij}^A\right)^+$ and post-multiplying by $\left(\boldsymbol{H}_{ji}^A\right)^+$,

$$\left(\boldsymbol{H}_{ij}^A\right)^+\left(\boldsymbol{H}_{ii}^A-\boldsymbol{H}_{ii}^C\right)\left(\boldsymbol{H}_{ji}^A\right)^+ = \boldsymbol{Z}_{jj}^B\left(\boldsymbol{I}_{jj}+\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B\right)^{-1} \tag{5}$$

which may be rewritten as

$$\boldsymbol{Z}_{jj}^B = \left(\left(\boldsymbol{H}_{ij}^A\right)^+\left(\boldsymbol{H}_{ii}^A-\boldsymbol{H}_{ii}^C\right)\left(\boldsymbol{H}_{ji}^A\right)^+\right)\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B + \left(\boldsymbol{H}_{ij}^A\right)^+\left(\boldsymbol{H}_{ii}^A-\boldsymbol{H}_{ii}^C\right)\left(\boldsymbol{H}_{ji}^A\right)^+ \tag{6}$$

Eq. (6) exhibits the "classic" format of the iterative methods for the solution of linear matrix equations , whose general form is the following one

$$\boldsymbol{X}^{(k+1)} = \boldsymbol{G}\boldsymbol{X}^{(k)} + \boldsymbol{P} \tag{7}$$

This process converges if and only if the spectral radius of the iteration matrix is less than 1 ($\rho(G) < 1$). From (4) one can use the other three elements and proceed similarly, obtaining three more different formulations for the dynamic stiffness

$$\boldsymbol{Z}_{jj}^B = \left(\left(\boldsymbol{H}_{ij}^A\right)^+\left(\boldsymbol{H}_{ij}^A-\boldsymbol{H}_{ij}^C\right)\left(\boldsymbol{H}_{jj}^A\right)^{-1}\right)\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B + \left(\boldsymbol{H}_{ij}^A\right)^+\left(\boldsymbol{H}_{ij}^A-\boldsymbol{H}_{ij}^C\right)\left(\boldsymbol{H}_{jj}^A\right)^{-1} \tag{8}$$

$$\boldsymbol{Z}_{jj}^B = \left(\left(\boldsymbol{H}_{jj}^A\right)^{-1}\left(\boldsymbol{H}_{ji}^A-\boldsymbol{H}_{ji}^C\right)\left(\boldsymbol{H}_{ji}^A\right)^+\right)\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B + \left(\boldsymbol{H}_{jj}^A\right)^{-1}\left(\boldsymbol{H}_{ji}^A-\boldsymbol{H}_{ji}^C\right)\left(\boldsymbol{H}_{ji}^A\right)^+ \tag{9}$$

$$\boldsymbol{Z}_{jj}^B = \left(\left(\boldsymbol{H}_{jj}^A\right)^{-1}\left(\boldsymbol{H}_{jj}^A-\boldsymbol{H}_{jj}^C\right)\left(\boldsymbol{H}_{jj}^A\right)^{-1}\right)\boldsymbol{H}_{jj}^A\boldsymbol{Z}_{jj}^B + \left(\boldsymbol{H}_{jj}^A\right)^{-1}\left(\boldsymbol{H}_{jj}^A-\boldsymbol{H}_{jj}^C\right)\left(\boldsymbol{H}_{jj}^A\right)^{-1} \tag{10}$$

Once Z^B is found, a regression using a least squares calculation is performed to find the stiffness and mass associated to each element, of the form $k-\omega^2 m$. One of the problems of this process is the shortage of frequencies where the spectral radius of the iteration matrix *G* is less than 1. The results of Z^B for eq. (10) are quite reasonable, in spite of considerably high perturbations in some frequency bands.

Analysis of the condition number

From eq. (10) is clear that both matrices *G* and *P* include the term $\left(\boldsymbol{H}_{jj}^A - \boldsymbol{H}_{jj}^C\right)$, responsible for some numerical instability. Thus, a function $r(\omega)$ is defined as

$$r(\omega) = \tau \cdot \ell n(cond(\boldsymbol{H}_{jj}^A - \boldsymbol{H}_{jj}^C)) \tag{11}$$

where τ is an amplifier of the condition effect. The local minima of $r(\omega)$ represent the frequency range where the problem is better conditioned. Thus, it is advisable to choose only those frequencies; $r(\omega)$ exhibits a great dispersion; so, to facilitate the choice of frequencies, a regression has been carried out with the *MatLab*® function:

$$R(\omega) = \mathrm{csaps}(r(\omega), v) \tag{12}$$

where v is a weighting parameter varying between 0 and 1. To determine the local minima one choses the frequency range where

$$\frac{dR(\omega)}{d\omega} < \varepsilon \quad \wedge \quad \frac{d^2R(\omega)}{d\omega^2} > 0 \tag{13}$$

Simulation study

Let us consider a coupling between two beams, connected by a joint element, as in Figure 2. Structure C is composed by A_1, B and A_2.

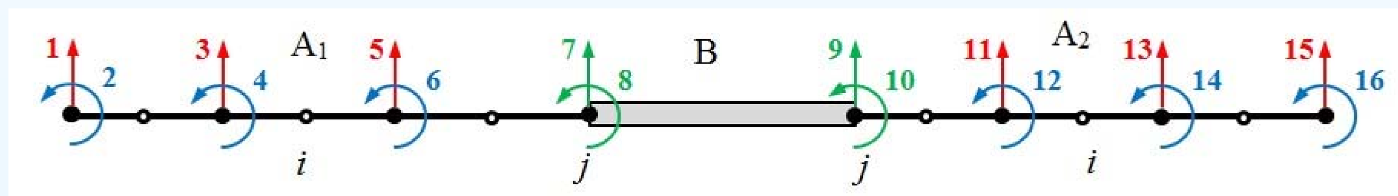

Figure 2 : Coupling of substructures A_1, B and A_2 .

To simulate the experimental errors, an amplitude independent error is added:

$$\tilde{H}_{pq}^C(\omega_k) = H_{pq}^C(\omega_k) + \frac{\gamma}{100}\cdot normrnd(\omega_k)\cdot \max\left(\left|H_{pq}^C(\omega)\right|\right) \tag{14}$$

where γ is the noise level (3% in this case) and $normrnd(\omega)$ is a *MatLab*® function that represents a normal distribution with zero mean and standard deviation 1.

Results

The chosen parameters are: $\tau = 30$, $v = 10^{-6}$ and $\varepsilon = 0{,}05$. Figure 3 represents $r(\omega)$, $R(\omega)$ and the chosen frequencies, identified by the vertical grey bands.

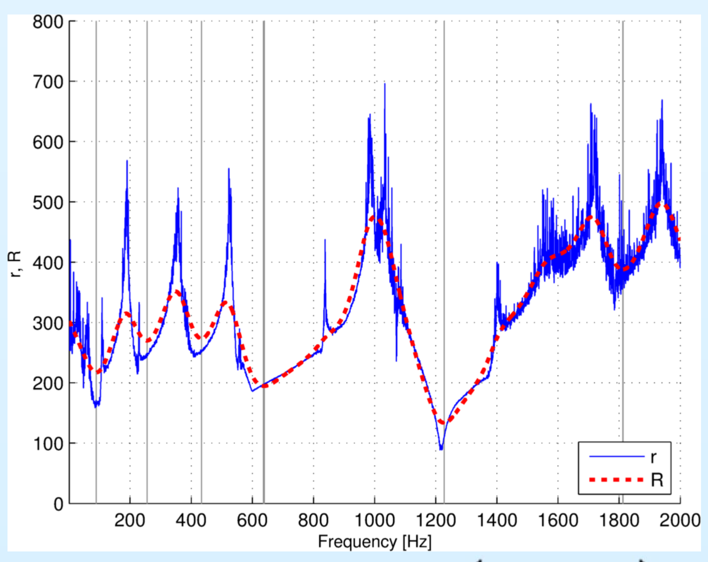

Figure 3: $r(\omega)$ and $R(\omega)$ of $\left(\boldsymbol{H}_{jj}^A - \boldsymbol{H}_{jj}^C\right)$

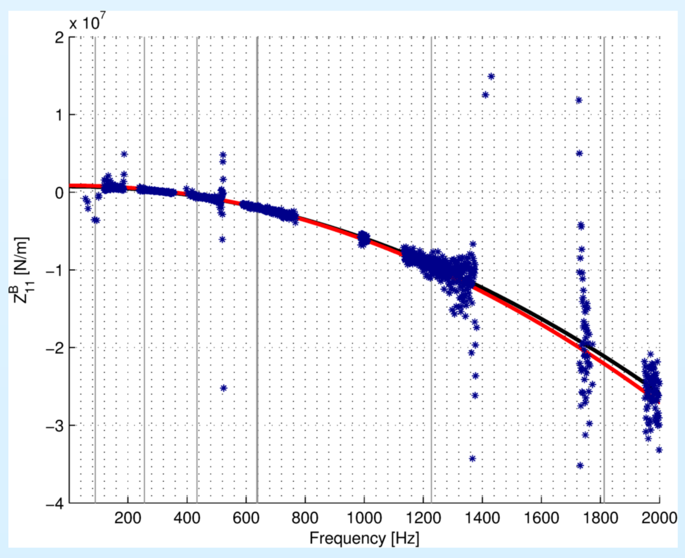

Figure 4: Z_{11}^B from eq.(10), with the indication of the frequency bands with minimum condition values

After the dynamic stiffness has been evaluated, with the calculation of the mass and stiffness matrices of substructure *B*, the receptance $\boldsymbol{H}^B = (\boldsymbol{K}^B - \omega^2\boldsymbol{M}^B)^{-1}$.

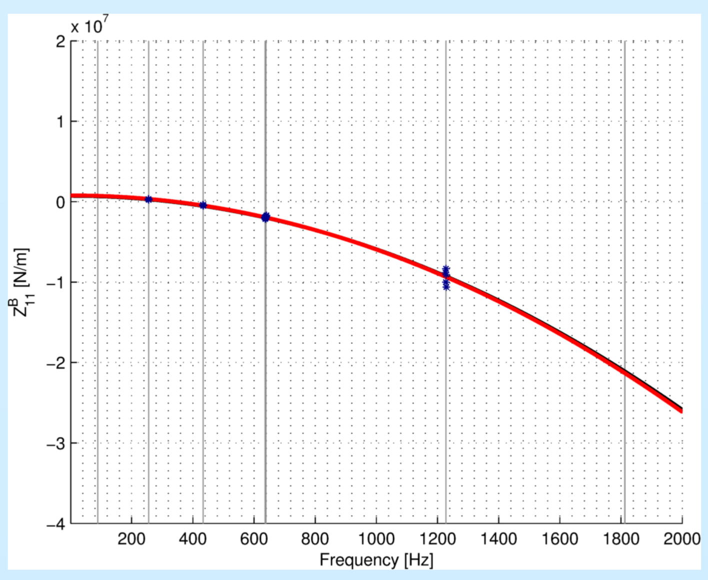

Figure 5: Z_{11}^B from eq.(10), with the frequency bands with minimum condition values

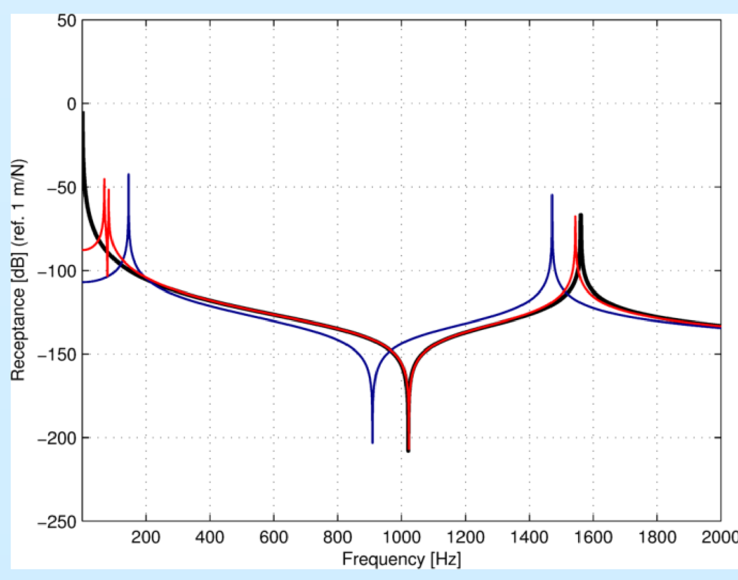

Figure 6: H_{11}^B ; blue: with all the points of fig. 4; red : with all the points of fig. 5; black: exact solution

Conclusions

An iterative method for the identification of joints has been presented; it does not need to invert any matrices, although it requires the responses at the joint co-ordinates. The identification of the frequencies where the condition number is minimum proved to be very effective, with significant improvements in the results.

Acknowledgments

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