ESTIMATION OF UNMEASURED FREQUENCY RESPONSE FUNCTIONS

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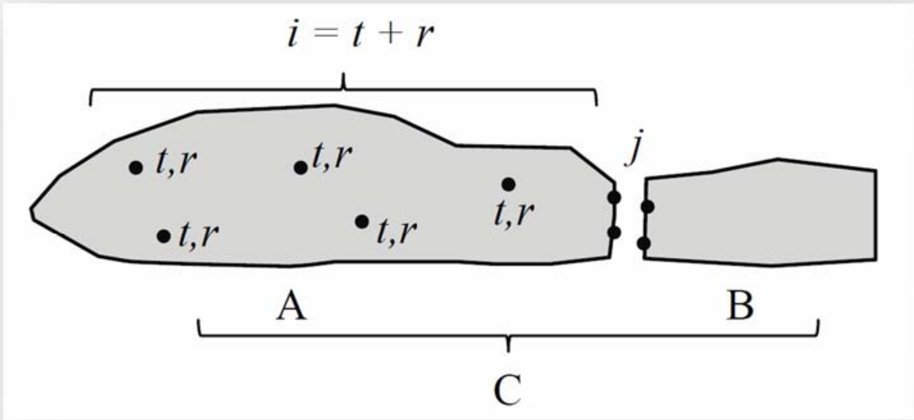

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Summary

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INTRODUCTION

THEORETICAL FORMULATION

- ***FRF* coupling** - regardless of the dynamic stiffness of B
  - Summary
- **Estimation of unmeasured FRFs**
  - Summary
- **Frequency-domain correlation criterion**

SIMULATION STUDIES

- **Numerical noise**
- **Quantification of the effect of the error**

CONCLUSIONS

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INTRODUCTION

- **Structural Uncoupling**
  - Modal Domain
    - **CMS**: **C**omponent **M**ode **S**ynthesis
    - Dual Craig–Bampton method
  - Mechanical Impedance
    - Mass matrix
    - Stiffness matrix
    - Damping matrix
  - Frequency Domain
    - **FBS**: **FRF - b**ased **s**ubstructuring

THEORETICAL FORMULATION

FRF **coupling** - regardless of the dynamic stiffness of B

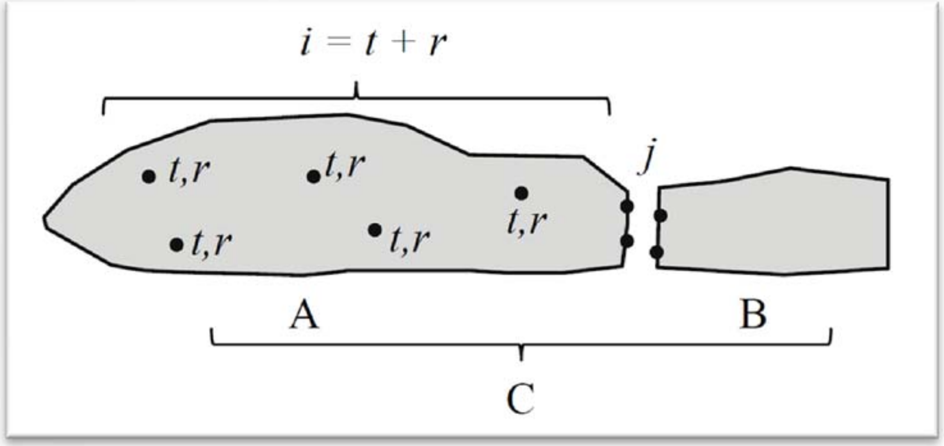

Figure 1

Substructure A is supposed to be easy to model numerically using finite element modeling.

The sum of the dynamic stiffness

$$\boldsymbol{Z}^C = \boldsymbol{Z}^A + \boldsymbol{Z}^B \quad (1)$$

- Co-ordinates *j* are coupling
- Co-ordinates *i* are the only sub-structure A, divided:
  - Co-ordinates *t* are experimentally measured
  - Co-ordinates *r* for some reason – have not been measured

The objective is therefore to evaluate the FRFs at co-ordinates *r* and *j* of structure C

THEORETICAL FORMULATION

***FRF* coupling** - regardless of the dynamic stiffness of B

Rearranging the second member of Eq. (1)

$$\boldsymbol{Z}^{C} = \boldsymbol{Z}^{A}\left(\boldsymbol{I} + \boldsymbol{H}^{A}\boldsymbol{Z}^{B}\right) \quad (2)$$

H^A is the FRF matrix of A, i.e., the inverse of the dynamic stiffness Z^A

Inverting both members

$$\boldsymbol{H}^{C} = \left(\boldsymbol{I} + \boldsymbol{H}^{A}\boldsymbol{Z}^{B}\right)^{-1}\boldsymbol{H}^{A} \quad (3)$$

Explicitly, in terms of the sub-matrices involving co-ordinates *i* and *j*

$$\begin{bmatrix} \boldsymbol{H}^{C}_{ii} & \boldsymbol{H}^{C}_{ij} \\ \boldsymbol{H}^{C}_{ji} & \boldsymbol{H}^{C}_{jj} \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I}_{ii} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{jj} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}^{A}_{ii} & \boldsymbol{H}^{A}_{ij} \\ \boldsymbol{H}^{A}_{ji} & \boldsymbol{H}^{A}_{jj} \end{bmatrix}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}^{B}_{jj} \end{bmatrix}\right]^{-1}\begin{bmatrix} \boldsymbol{H}^{A}_{ii} & \boldsymbol{H}^{A}_{ij} \\ \boldsymbol{H}^{A}_{ji} & \boldsymbol{H}^{A}_{jj} \end{bmatrix} \quad (4)$$

THEORETICAL FORMULATION

***FRF* coupling** - regardless of the dynamic stiffness of B

Simplifying Eq. (4),

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{ii} & -\boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1} \\ \boldsymbol{0} & \left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (5)$$

from which

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A} \\ \left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A} & \left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (6)$$

To facilitate the development of the formulation it is important to note that

$$\boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{I}_{jj} + \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1} = \boldsymbol{Z}_{jj}^{B}\left(\left(\boldsymbol{Z}_{jj}^{B}\right)^{-1}\left(\boldsymbol{Z}_{jj}^{B} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)\right)^{-1} \quad (7)$$

$$= \boldsymbol{Z}_{jj}^{B}\left(\boldsymbol{Z}_{jj}^{B} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{Z}_{jj}^{B} = \boldsymbol{Z}_{jj}^{B}\left(\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)\boldsymbol{Z}_{jj}^{B}\right)^{-1}\boldsymbol{Z}_{jj}^{B} = \left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}$$

THEORETICAL FORMULATION

FRF **coupling** - regardless of the dynamic stiffness of B

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \left[\begin{bmatrix} \boldsymbol{I}_{ii} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{jj} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{jj}^{B} \end{bmatrix} \right]^{-1} \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (4)$$

Eq. (4) can also be written as

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} + \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Z}_{jj}^{B} \end{bmatrix} \begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} \quad (8)$$

or simply

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \\ \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \end{bmatrix} \quad (9)$$

THEORETICAL FORMULATION

FRF **coupling** - regardless of the dynamic stiffness of B

from Eqs. (6), (7) and (9):

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (10)$$

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} \quad (11)$$

Rearranging:

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (12)$$

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{C} \quad (13)$$

Rearranging Eq. (12),

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \quad (14)$$

Rearranging Eq. (14),

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\left[\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right)\right] \quad (15)$$

Substituting Eq. (15) in Eq. (13),

$$\boxed{\boldsymbol{H}_{ji}^{C} = \boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right)} \quad (16)$$

THEORETICAL FORMULATION

FRF **coupling** - regardless of the dynamic stiffness of B

Similar process, from Eqs. (6), (7) and (9):

$$\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A} \quad (17)$$

$$\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \quad (18)$$

Rearranging:

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right) = \left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A} \quad (19)$$

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{C} \quad (20)$$

Rearranging Eq. (19),

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right) + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A} \quad (21)$$

Rearranging Eq. (21),

$$\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right) = \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\boldsymbol{H}_{ij}^{C} \quad (22)$$

Substituting Eq. (20) in Eq. (22),

$$\boxed{\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\boldsymbol{H}_{ij}^{C}} \quad (23)$$

THEORETICAL FORMULATION

FRF **coupling** - regardless of the dynamic stiffness of B

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{ji}^{A} \tag{10}$$

$$\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A} \tag{17}$$

Rearranging Eq. (17):

$$\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right)\left(\boldsymbol{H}_{jj}^{A}\right)^{-1} = \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{I}_{jj} + \boldsymbol{Z}_{jj}^{B}\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{Z}_{jj}^{B} \tag{24}$$

Substituting Eq. (24) in Eq. (10) leads to

$$\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C} = \left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right)\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{H}_{ji}^{A} \tag{25}$$

Transposing (25), one finally obtains

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{ji}^{C}\right) \tag{26}$$

THEORETICAL FORMULATION

***FRF* coupling** - regardless of the dynamic stiffness of B

Summary

One can now determine H^C without needing to know anything about substructure *B*, using the following equations:

$$\boldsymbol{H}_{ji}^{C} = \boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) \tag{16}$$

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{ji}^{C}\right) \tag{26}$$

$$\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\boldsymbol{H}_{ij}^{C} \tag{23}$$

THEORETICAL FORMULATION

Estimation of unmeasured FRFs

Co-ordinates *i* of structure C are formed by co-ordinates *t* and *r*.
Matrix H^C is symmetric.

Only possible to determine the sub-matrix H^C_{tt} experimentally,

$$\boldsymbol{H}^C = \begin{bmatrix} \boldsymbol{H}^C_{ii} & \boldsymbol{H}^C_{ij} \\ \boldsymbol{H}^C_{ji} & \boldsymbol{H}^C_{jj} \end{bmatrix} \qquad i = t + r \quad \rightarrow \quad \boldsymbol{H}^C = \begin{bmatrix} \boldsymbol{H}^C_{tt} & \boldsymbol{H}^C_{tr} & \boldsymbol{H}^C_{tj} \\ \boldsymbol{H}^C_{rt} & \boldsymbol{H}^C_{rr} & \boldsymbol{H}^C_{rj} \\ \boldsymbol{H}^C_{jt} & \boldsymbol{H}^C_{jr} & \boldsymbol{H}^C_{jj} \end{bmatrix} \tag{27}$$

$$\boldsymbol{H}^C_{ji} = \boldsymbol{H}^A_{ji} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{ij}\right)^{+}\left(\boldsymbol{H}^A_{ii} - \boldsymbol{H}^C_{ii}\right) \tag{16}$$

From Eq. (16), considering only translational co-ordinates, i.e., *ii* = *tt*

$$\boldsymbol{H}^C_{jt} = \boldsymbol{H}^A_{jt} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^{+}\left(\boldsymbol{H}^A_{tt} - \boldsymbol{H}^C_{tt}\right) \tag{28}$$

THEORETICAL FORMULATION

Estimation of unmeasured FRFs

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{ji}^{C}\right) \quad (26)$$

$$\boldsymbol{H}_{jt}^{C} = \boldsymbol{H}_{jt}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{tj}^{A}\right)^{+}\left(\boldsymbol{H}_{tt}^{A} - \boldsymbol{H}_{tt}^{C}\right) \quad (28)$$

From Eqs. (26) and (28), relating rotational to translational co-ordinates such that $ii = rt$, it follows that

$$\boldsymbol{H}_{rt}^{C} = \boldsymbol{H}_{rt}^{A} - \boldsymbol{H}_{rj}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{jt}^{A} - \boldsymbol{H}_{jt}^{C}\right) \quad (29)$$

$$\boldsymbol{H}_{ji}^{C} = \boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right) \quad (16)$$

From Eqs. (16) and (29), relating translational to rotational co-ordinates such that $ii = tr$, the result is

$$\boldsymbol{H}_{jr}^{C} = \boldsymbol{H}_{jr}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{tj}^{A}\right)^{+}\left(\boldsymbol{H}_{tr}^{A} - \boldsymbol{H}_{tr}^{C}\right) \quad (30)$$

THEORETICAL FORMULATION

Estimation of unmeasured FRFs

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{ji}^{A} - \boldsymbol{H}_{ji}^{C}\right) \qquad (26)$$

$$\boldsymbol{H}_{jr}^{C} = \boldsymbol{H}_{jr}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{tj}^{A}\right)^{+}\left(\boldsymbol{H}_{tr}^{A} - \boldsymbol{H}_{tr}^{C}\right) \qquad (30)$$

From Eqs. (26) and (30), considering only rotational co-ordinates, i.e., $ii = rr$, one has,

$$\boldsymbol{H}_{rr}^{C} = \boldsymbol{H}_{rr}^{A} - \boldsymbol{H}_{rj}^{A}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\left(\boldsymbol{H}_{jr}^{A} - \boldsymbol{H}_{jr}^{C}\right) \qquad (31)$$

$$\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ij}^{A}\right)^{+}\boldsymbol{H}_{ij}^{C} \qquad (23)$$

$$\boldsymbol{H}_{jt}^{C} = \boldsymbol{H}_{jt}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{tj}^{A}\right)^{+}\left(\boldsymbol{H}_{tt}^{A} - \boldsymbol{H}_{tt}^{C}\right) \qquad (28)$$

Finally, Eqs. (23) and (28), for $i = t$, yield

$$\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{tj}^{A}\right)^{+}\boldsymbol{H}_{tj}^{C} \qquad (28)$$

THEORETICAL FORMULATION

Estimation of unmeasured FRFs

Summary
- H^A are determined numerically by *FEM*
- H^C_{tt} known responses measured experimentally

$$\boldsymbol{H}^C_{jt} = \boldsymbol{H}^A_{jt} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\left(\boldsymbol{H}^A_{tt} - \boldsymbol{H}^C_{tt}\right) \quad (28)$$

$$\boldsymbol{H}^C_{rt} = \boldsymbol{H}^A_{rt} - \boldsymbol{H}^A_{rj}\left(\boldsymbol{H}^A_{jj}\right)^{-1}\left(\boldsymbol{H}^A_{jt} - \boldsymbol{H}^C_{jt}\right) \quad (29)$$

$$\boldsymbol{H}^C_{jr} = \boldsymbol{H}^A_{jr} - \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\left(\boldsymbol{H}^A_{tr} - \boldsymbol{H}^C_{tr}\right) \quad (30)$$

$$\boldsymbol{H}^C_{rr} = \boldsymbol{H}^A_{rr} - \boldsymbol{H}^A_{rj}\left(\boldsymbol{H}^A_{jj}\right)^{-1}\left(\boldsymbol{H}^A_{jr} - \boldsymbol{H}^C_{jr}\right) \quad (31)$$

$$\boldsymbol{H}^C_{jj} = \boldsymbol{H}^A_{jj}\left(\boldsymbol{H}^A_{tj}\right)^+\boldsymbol{H}^C_{tj} \quad (32)$$

THEORETICAL FORMULATION

Frequency-domain correlation criterion

A simple visual comparison of the *FRFs* calculated numerically ($H_A(\omega)$) with those obtained experimentally ($H_X(\omega)$) only provides a qualitative idea of the goodness of the results.

LAC - *Local Amplitude Criterion*

$$LAC_{ij}(\omega) = \frac{2\left|H_{Xij}(\omega)^* \cdot H_{Aij}(\omega)\right|}{\left(H_{Xij}(\omega)^* \cdot H_{Xij}(\omega)\right) + \left(H_{Aij}(\omega)^* \cdot H_{Aij}(\omega)\right)}$$

where i and j are the response and excitation co-ordinates, respectively.

$H_{Aij}(\omega)$ is the *FRF* obtained numerically.

$H_{Xij}(\omega)$ is the *FRF* obtained experimentally.

$$0 < LAC_{ij}(\omega) \leq 1$$

Average ***LAC***

$$\overline{LAC_{ij}} = \frac{1}{N}\sum_{k=1}^{N} LAC_{ij}(\omega_k)$$

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SIMULATION STUDIES

To validate the proposed method, four numerical examples of coupled structures are presented and illustrated in Figs. 2, 3, 4 and 5.

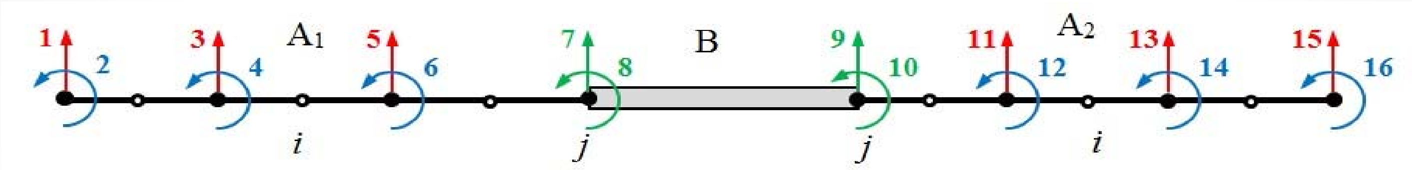

Figure 2: Case 1

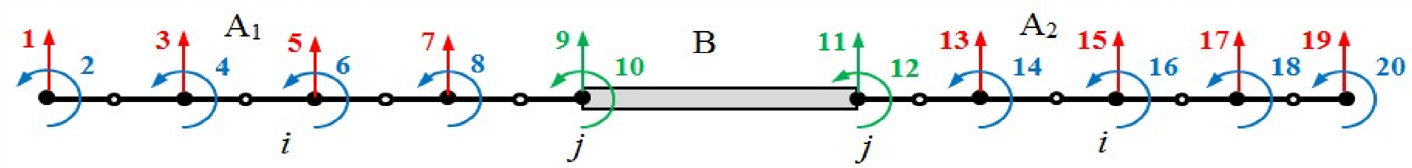

Figure 3: Case 2

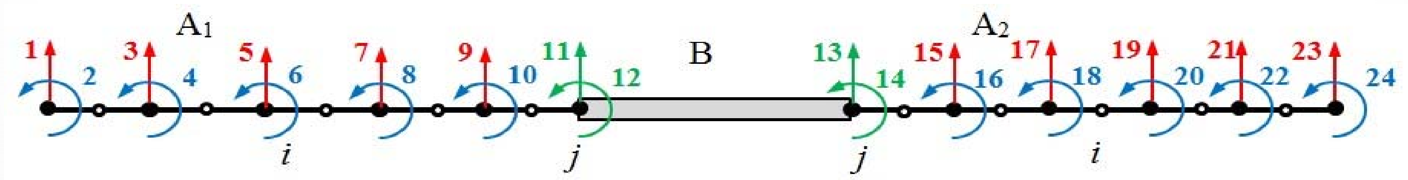

Figure 4: Case 3

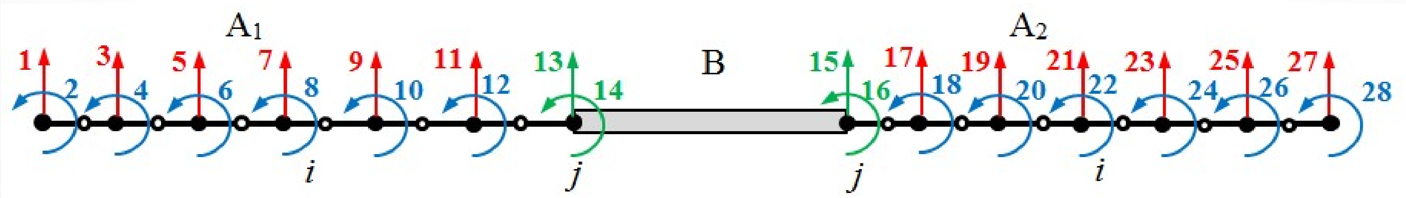

Figure 5: Case 4

Using the *FEM* with beam elements with 4 degrees of freedom.

SIMULATION STUDIES

In Figs. 2, 3, 4 and 5,
the translational co-ordinates *t* (known) are the ones in red colour;
in blue, one has the rotational co-ordinates *r* (unknown);
the joint co-ordinates *j* (unknown) are represented in green.

Table1: Characteristics of the components of the beam

Beam	Length	Width	Thickness	E	□
A_1	270 mm	30 mm	5 mm	194 GPa	7562 Kg/m^3
B	200 mm	30 mm	10 mm	194 GPa	7562 Kg/m^3
A_2	370 mm	30 mm	5 mm	194 GPa	7562 Kg/m^3

Numerical noise

Numerical random error is independent of the amplitude.

$$\tilde{H}_{tt}(\omega_k) = H_{tt}(\omega_k) + \frac{\gamma}{100} \cdot normrnd(\omega_k) \cdot \max\left(\left|H_{tt}(\omega)\right|\right)$$

where □ is the noise level in percentage and *normrnd*(ω) is a normal distribution with zero mean and standard deviation equal to one.
A noise level of 3% has been added.

Quantification of the effect of the error

Figures 6, 7, 8 and 9 show the averaged *LAC* for each matrix H^C

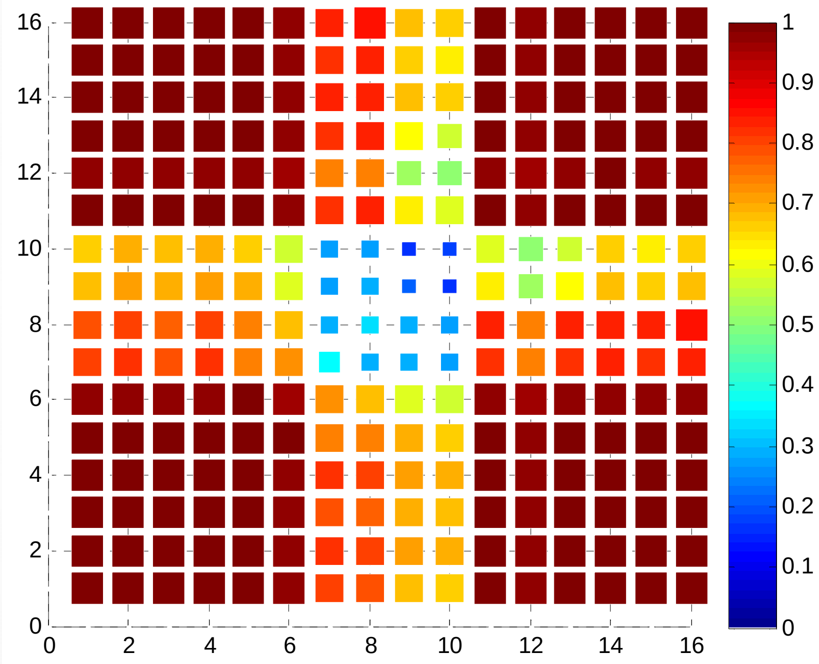

Figure 6: Averaged *LAC* - Case 1

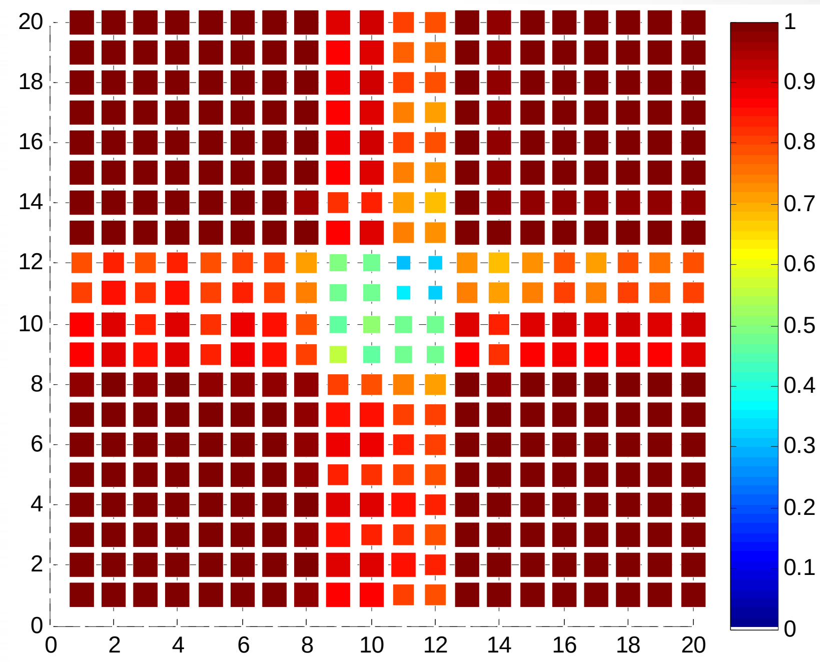

Figure 7: Averaged *LAC* - Case 2

Quantification of the effect of the error

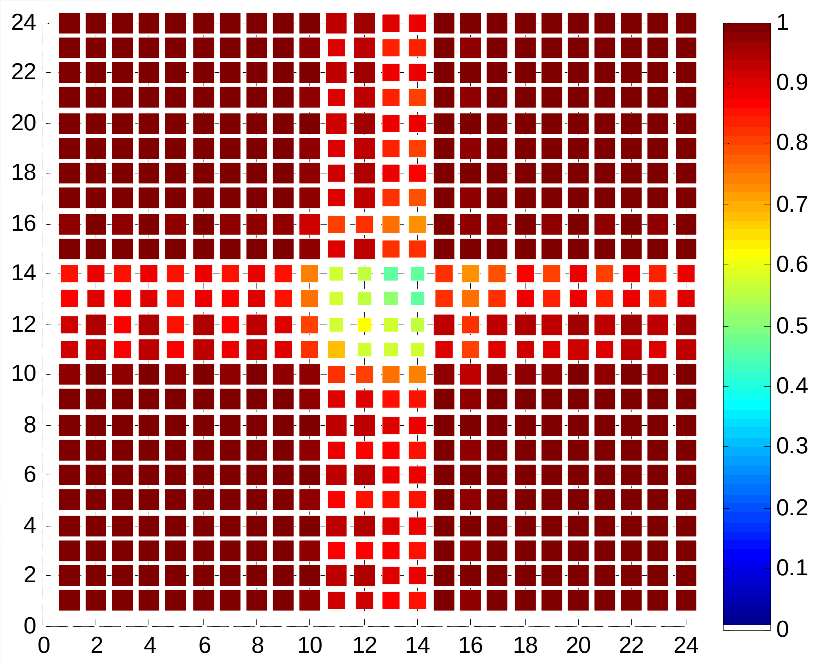

Figure 8: Averaged *LAC* - Case 3

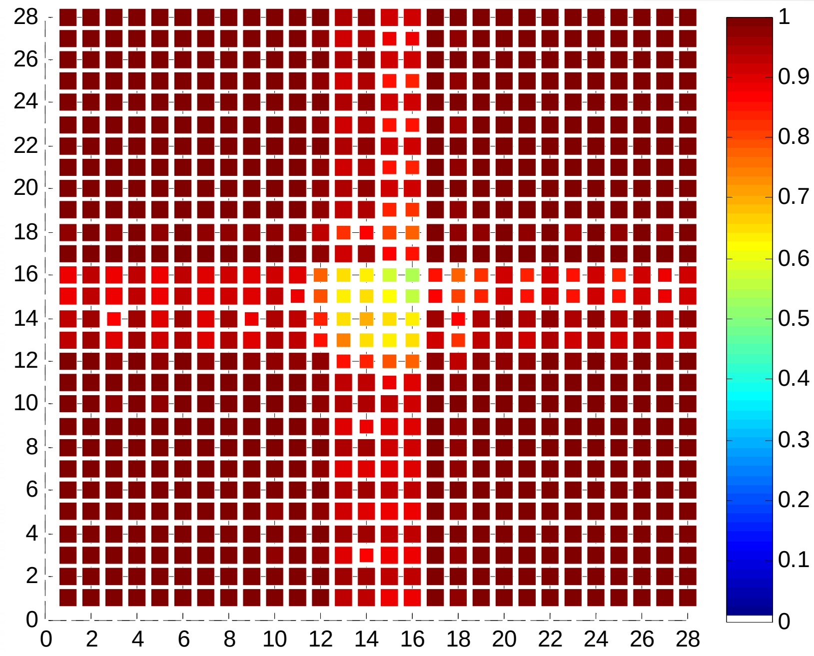

Figure 9: Averaged *LAC* - Case 4

The values relating co-ordinates *i* show a very good correlation. However, the results involving co-ordinates *j* show a poor correlation. This effect is more significant for correlations between the co-ordinates *j* themselves.

There are considerable improvements if the number of co-ordinates *i* to be measured increases, as it can be observed when comparing Fig. 6 with Fig. 9.

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Quantification of the effect of the error

Figures 10 and 11 present two of the estimated *FRFs* of Case 1 against their theoretical values.

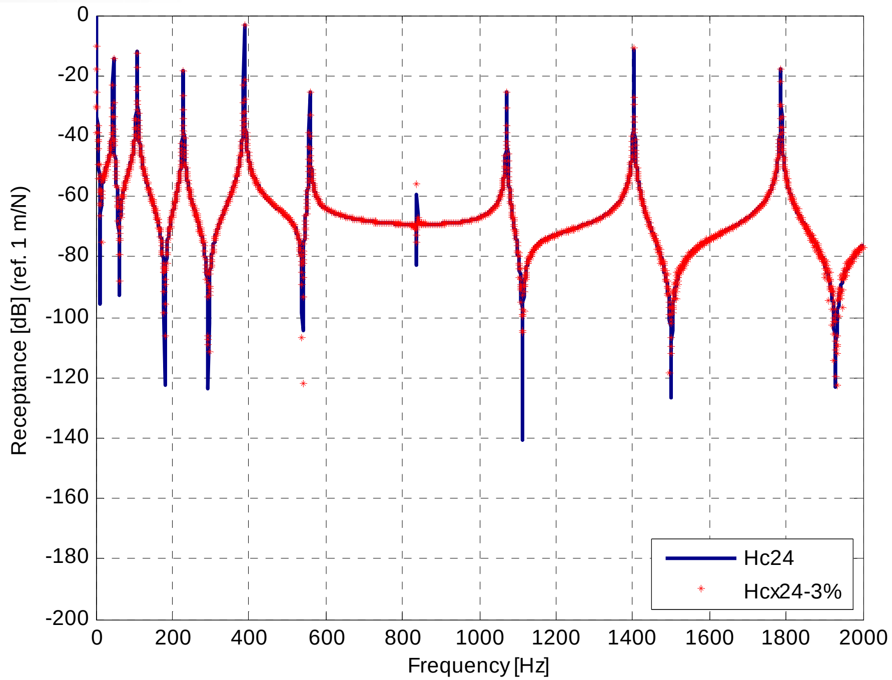

Figure 10: H_{24}^{C} - Case 1

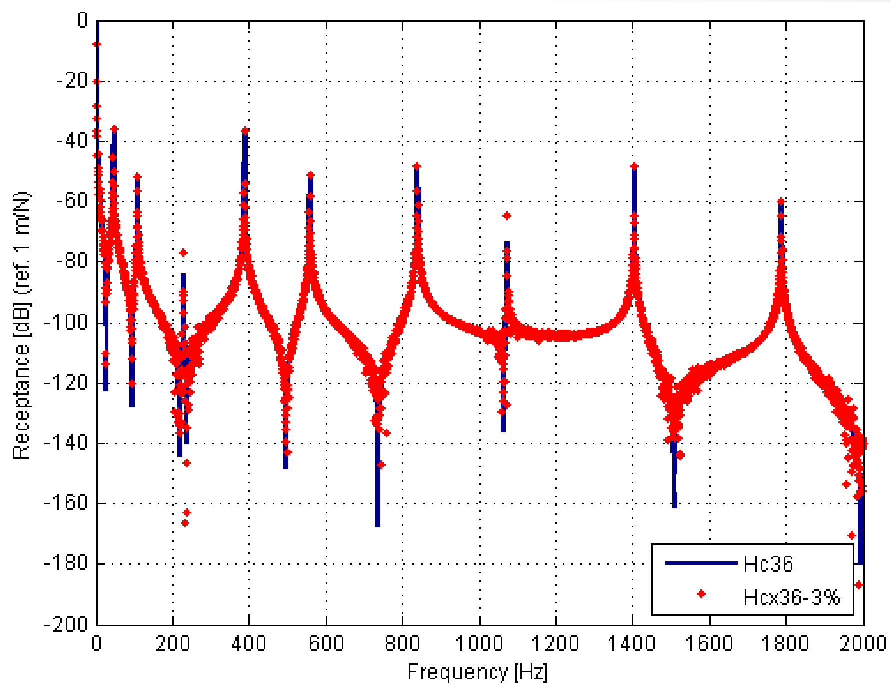

Figure 11: H_{36}^{C} - Case 1

Only some disturbances are noticed at the anti-resonances.

CONCLUSIONS

- In this paper a new methodology for estimating unmeasured *FRFs* has been presented, based upon classical formulations of *FRF* coupling and uncoupling of substructures; the process implies the knowledge of an analytical or numerical model of part of the structure and the measurement of translational *FRFs*.

- This methodology can be used to estimate *FRFs* involving rotational degrees-of-freedom, often difficult or impossible to measure.

- Some numerical examples have been presented to evaluate the performance of the technique and some noise has been added to simulate real data. Not surprisingly, it has been shown that an increase in the number of measured translational degrees-of-freedom improves the estimation of the unmeasured *FRFs*.

- The results are very promising for future applications in real experimental cases.

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Thank You!

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