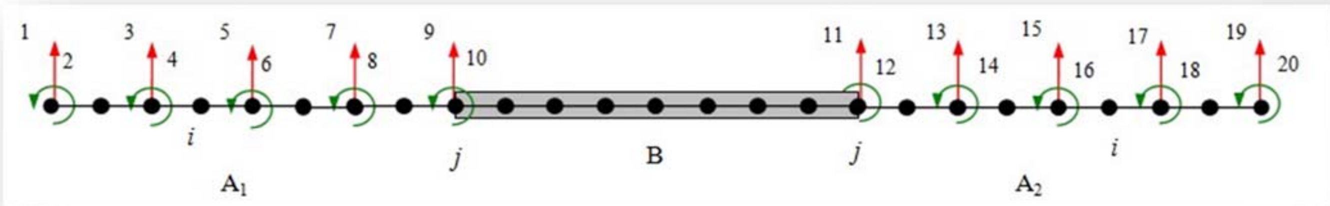
Uncoupling Techniques for the Dynamic Characterization of Sub-Structures



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Summary

INTRODUCTION

THEORETICAL FORMULATION

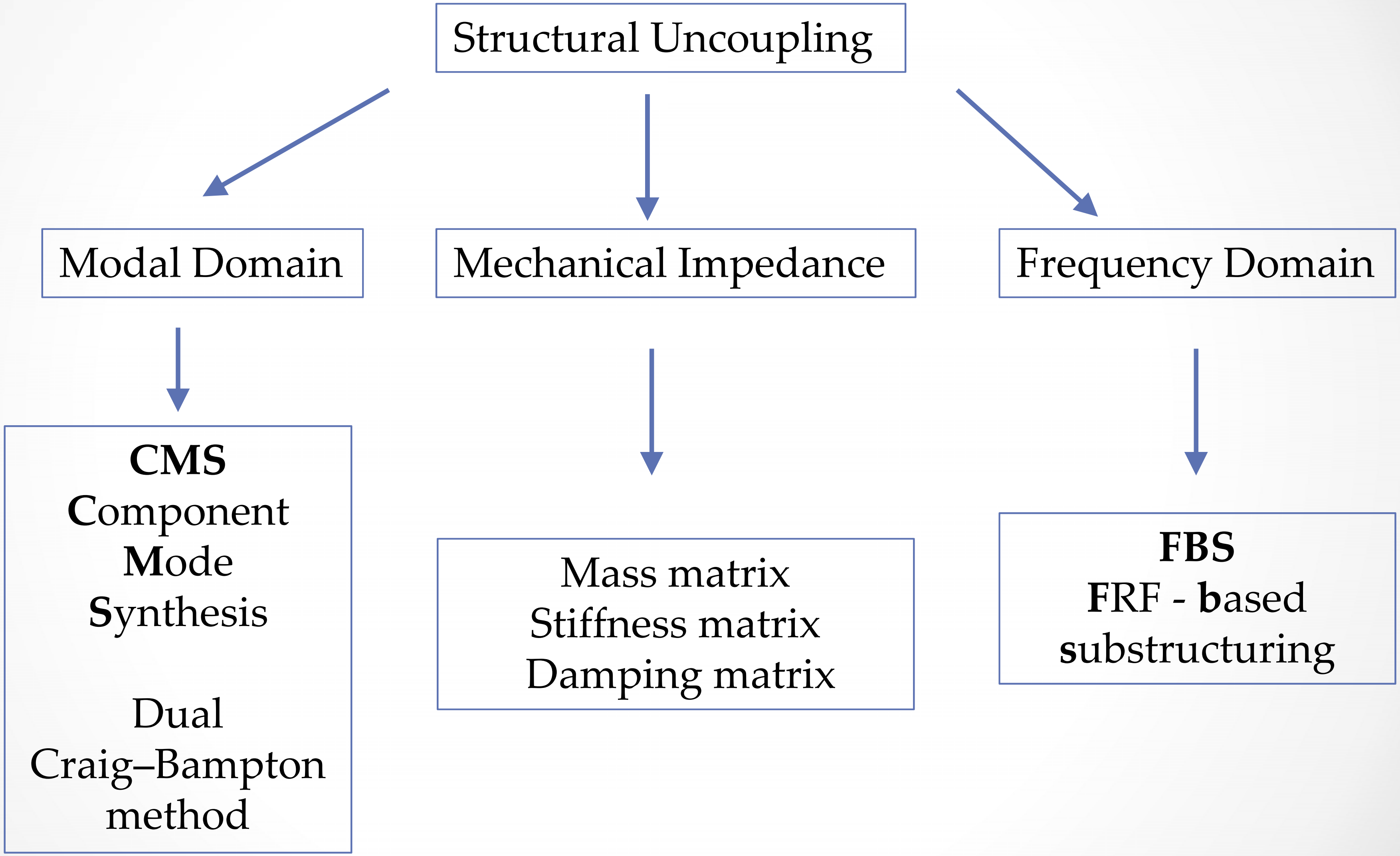
- ***FRF* coupling**
- ***FRF* uncoupling**
  - Without the use of co-ordinates j
  - Using only the coordinates of the joint
  - Using coordinates i and j
- **Summary**

SIMULATION STUDIES

- **Choice of the formulation**
- **Strategies to improve the results**
  - Adding mass to sub-structure A
  - Adding mass to sub-structure B
- **Coupling**

CONCLUSIONS

INTRODUCTION



THEORETICAL FORMULATION

FRF coupling

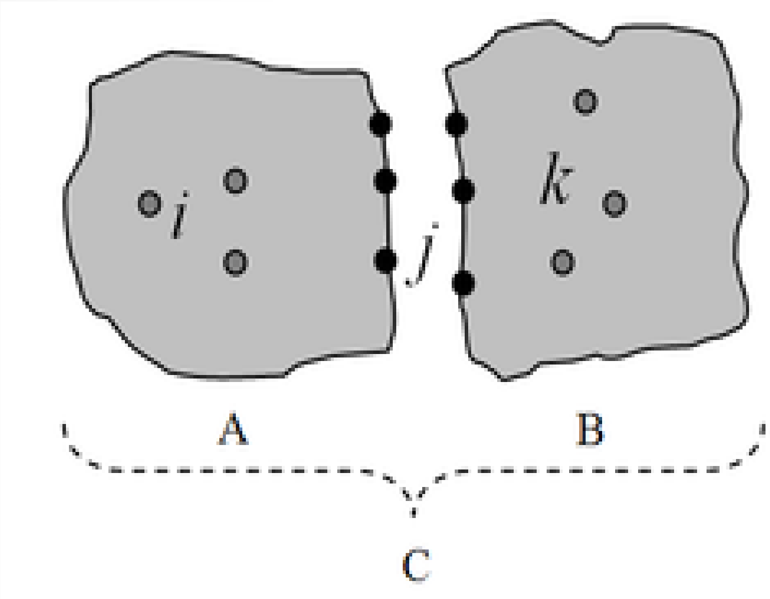


Fig. 1

Coordinates i are the only sub-structure A

Coordinates k are the only sub-structure B

Coordinate j are coupling

Equilibrium of forces and the compatibility conditions of displacements

$$\boldsymbol{f}_i^A + \boldsymbol{f}_i^B = \boldsymbol{f}_i^C \qquad \boldsymbol{x}_j^A = \boldsymbol{x}_j^B = \boldsymbol{x}_j^C$$

THEORETICAL FORMULATION

FRF coupling

Receptance matrices are defined $\boldsymbol{X} = \boldsymbol{H}\boldsymbol{F}$

Receptance matrices for sub-structures *A* and *B* and for structure *C*

$$\boldsymbol{H}^A = \begin{bmatrix} \boldsymbol{H}_{ii}^A & \boldsymbol{H}_{ij}^A \\ \boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A \end{bmatrix} \qquad \boldsymbol{H}^B = \begin{bmatrix} \boldsymbol{H}_{jj}^B & \boldsymbol{H}_{jk}^B \\ \boldsymbol{H}_{kj}^B & \boldsymbol{H}_{kk}^B \end{bmatrix} \qquad \boldsymbol{H}^C = \begin{bmatrix} \boldsymbol{H}_{ii}^C & \boldsymbol{H}_{ij}^C & \boldsymbol{H}_{ik}^C \\ \boldsymbol{H}_{ji}^C & \boldsymbol{H}_{jj}^C & \boldsymbol{H}_{jk}^C \\ \boldsymbol{H}_{ki}^C & \boldsymbol{H}_{kj}^C & \boldsymbol{H}_{kk}^C \end{bmatrix}$$

Using equilibrium equations and compatibility conditions are

$$\boldsymbol{H}^C = \left(\begin{bmatrix} \begin{bmatrix} \boldsymbol{H}_{ii}^A & \boldsymbol{H}_{ij}^A \\ \boldsymbol{H}_{ji}^A & \boldsymbol{H}_{jj}^A \end{bmatrix}^{-1} & \begin{matrix} \boldsymbol{0} \\ \boldsymbol{0} \end{matrix} \\ \begin{matrix} \boldsymbol{0} & \boldsymbol{0} \end{matrix} & \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} & \begin{matrix} \boldsymbol{0} & \boldsymbol{0} \end{matrix} \\ \begin{matrix} \boldsymbol{0} \\ \boldsymbol{0} \end{matrix} & \begin{bmatrix} \boldsymbol{H}_{jj}^B & \boldsymbol{H}_{jk}^B \\ \boldsymbol{H}_{kj}^B & \boldsymbol{H}_{kk}^B \end{bmatrix}^{-1} \end{bmatrix} \right)^{-1}$$

high computational effort

THEORETICAL FORMULATION

FRF coupling

Alternative formulation is used by Skingle

$$\mathbf{H}^{C}=\begin{bmatrix}\mathbf{H}_{ii}^{A} & \mathbf{H}_{ij}^{A} & \mathbf{0}\\ \mathbf{H}_{ji}^{A} & \mathbf{H}_{jj}^{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{H}_{kk}^{B}\end{bmatrix}-\begin{bmatrix}\mathbf{H}_{ij}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{ji}^{A} & \mathbf{H}_{ij}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jj}^{A} & -\mathbf{H}_{ij}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jk}^{B}\\ \mathbf{H}_{jj}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{ji}^{A} & \mathbf{H}_{jj}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jj}^{A} & -\mathbf{H}_{jj}^{A}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jk}^{B}\\ -\mathbf{H}_{kj}^{B}\mathbf{H}_{jj}^{-1}\mathbf{H}_{ji}^{A} & -\mathbf{H}_{kj}^{B}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jj}^{A} & \mathbf{H}_{kj}^{B}\mathbf{H}_{jj}^{-1}\mathbf{H}_{jk}^{B}\end{bmatrix}$$

Where $\mathbf{H}_{jj}=\mathbf{H}_{jj}^{A}+\mathbf{H}_{jj}^{B}$

We can simplify

$$\begin{bmatrix}\mathbf{H}_{ii}^{C} & \mathbf{H}_{ij}^{C} & \mathbf{H}_{ik}^{C}\\ \mathbf{H}_{ji}^{C} & \mathbf{H}_{jj}^{C} & \mathbf{H}_{jk}^{C}\\ \mathbf{H}_{ki}^{C} & \mathbf{H}_{kj}^{C} & \mathbf{H}_{kk}^{C}\end{bmatrix}=\begin{bmatrix}\mathbf{H}_{ii}^{A} & \mathbf{H}_{ij}^{A} & \mathbf{0}\\ \mathbf{H}_{ji}^{A} & \mathbf{H}_{jj}^{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{H}_{kk}^{B}\end{bmatrix}-\begin{bmatrix}\mathbf{H}_{ij}^{A}\\ \mathbf{H}_{jj}^{A}\\ -\mathbf{H}_{kj}^{B}\end{bmatrix}\left(\mathbf{H}_{jj}^{A}+\mathbf{H}_{jj}^{B}\right)^{-1}\begin{bmatrix}\mathbf{H}_{ji}^{A} & \mathbf{H}_{jj}^{A} & -\mathbf{H}_{jk}^{B}\end{bmatrix}$$

FRF **uncoupling**

If our joint is defined as sub-structure *B*, one has co-ordinates *i* and *j*, whereas co-ordinates *k* (internal to *B*) do not play a role,

$$\begin{bmatrix} \boldsymbol{H}_{ii}^{C} & \boldsymbol{H}_{ij}^{C} \\ \boldsymbol{H}_{ji}^{C} & \boldsymbol{H}_{jj}^{C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{H}_{ii}^{A} & \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix} - \begin{bmatrix} \boldsymbol{H}_{ij}^{A} \\ \boldsymbol{H}_{jj}^{A} \end{bmatrix} \left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B} \right)^{-1} \begin{bmatrix} \boldsymbol{H}_{ji}^{A} & \boldsymbol{H}_{jj}^{A} \end{bmatrix}$$

That there are **three** possibilities for the evaluation of $\boldsymbol{H}_{jj}^{B}$

Without the use of co-ordinates *j*

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A} \left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B} \right)^{-1} \boldsymbol{H}_{ji}^{A}$$

Using only
the coordinates of the joint *j*

$$\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A} - \boldsymbol{H}_{jj}^{A} \left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B} \right)^{-1} \boldsymbol{H}_{jj}^{A}$$

Using coordinates *i* and *j*

$$\boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{A} \left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B} \right)^{-1} \boldsymbol{H}_{jj}^{A}$$

THEORETICAL FORMULATION

FRF **uncoupling** - Without the use of co-ordinates *j*

$$\boldsymbol{H}_{ii}^{C} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A}$$

Rearranging $$\boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A} = \boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}$$

Generalizing to the case where *i* might be different from *j* (in fact, $i \geq j$), one pre-multiplies equation by an arbitrary matrix $\boldsymbol{W}_{ji}$ and post-multiplies it by $\boldsymbol{W}_{ij}$:

$$\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{ji}^{A}\boldsymbol{W}_{ij} = \boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right)\boldsymbol{W}_{ij}$$

Rearranging

$$\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1} = \left(\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\right)^{-1}\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ii}^{A} - \boldsymbol{H}_{ii}^{C}\right)\boldsymbol{W}_{ij}\left(\boldsymbol{H}_{ji}^{A}\boldsymbol{W}_{ij}\right)^{-1}$$

THEORETICAL FORMULATION

FRF **uncoupling** - Without the use of co-ordinates *j*

$$\left(\boldsymbol{H}_{jj}^{A}+\boldsymbol{H}_{jj}^{B}\right)^{-1}=\left(\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\right)^{-1}\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ii}^{A}-\boldsymbol{H}_{ii}^{C}\right)\boldsymbol{W}_{ij}\left(\boldsymbol{H}_{ji}^{A}\boldsymbol{W}_{ij}\right)^{-1}$$

This is only possible if $i \geq j$ not the other way around (which certainly is not a common case).

$$\boldsymbol{H}_{jj}^{B}=\boldsymbol{H}_{ji}^{A}\boldsymbol{W}_{ij}\left(\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ii}^{A}-\boldsymbol{H}_{ii}^{C}\right)\boldsymbol{W}_{ij}\right)^{-1}\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{jj}^{A}$$

The question is which matrix $\boldsymbol{W}_{ij}$ to use.

Probably the most logical one is to use $\boldsymbol{H}_{ij}^{A}$

$$\boldsymbol{H}_{jj}^{B}=\boldsymbol{H}_{ji}^{A}\boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{ji}^{A}\left(\boldsymbol{H}_{ii}^{A}-\boldsymbol{H}_{ii}^{C}\right)\boldsymbol{H}_{ij}^{A}\right)^{-1}\boldsymbol{H}_{ji}^{A}\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{jj}^{A}$$

THEORETICAL FORMULATION

FRF **uncoupling** - Using only the coordinates of the joint *j*

$$\boldsymbol{H}_{jj}^{C} = \boldsymbol{H}_{jj}^{A} - \boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A}$$

Solving this equation with respect to $\boldsymbol{H}_{jj}^{B}$ it follows that

$$\boldsymbol{H}_{jj}^{B} = \left(\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{jj}^{A} - \boldsymbol{H}_{jj}^{C}\right)^{-1} - \boldsymbol{I}_{jj}\right)\boldsymbol{H}_{jj}^{A}$$

Ambrogio obtains an alternative formulation (we can be derived the next equation from last):

$$\boldsymbol{H}_{jj}^{B} = \left(\boldsymbol{I}_{jj} - \boldsymbol{H}_{jj}^{C}\left(\boldsymbol{H}_{jj}^{A}\right)^{-1}\right)^{-1}\boldsymbol{H}_{jj}^{C}$$

FRF **uncoupling** - Using coordinates *i* and *j*

$$\boldsymbol{H}_{ij}^{C} = \boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A}$$

Rearranging

$$\boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A} = \boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}$$

Using once again arbitrary matrices, now $\boldsymbol{W}_{ji}$ and $\boldsymbol{W}_{jj}$ with $i \geq j$

$$\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1}\boldsymbol{H}_{jj}^{A}\boldsymbol{W}_{jj} = \boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right)\boldsymbol{W}_{jj}$$

Rearranging

$$\left(\boldsymbol{H}_{jj}^{A} + \boldsymbol{H}_{jj}^{B}\right)^{-1} = \left(\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\right)^{-1}\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ij}^{A} - \boldsymbol{H}_{ij}^{C}\right)\boldsymbol{W}_{jj}\left(\boldsymbol{H}_{jj}^{A}\boldsymbol{W}_{jj}\right)^{-1}$$

THEORETICAL FORMULATION

***FRF* uncoupling** - Using coordinates *i* and *j*

$$\left(\boldsymbol{H}_{jj}^{A}+\boldsymbol{H}_{jj}^{B}\right)^{-1}=\left(\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}\right)^{-1}\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{ij}^{C}\right)\boldsymbol{W}_{jj}\left(\boldsymbol{H}_{jj}^{A}\boldsymbol{W}_{jj}\right)^{-1}$$

Solving in order to $\boldsymbol{H}_{jj}^{B}$

$$\boldsymbol{H}_{jj}^{B}=\boldsymbol{H}_{jj}^{A}\boldsymbol{W}_{jj}\left(\boldsymbol{W}_{ji}\left(\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{ij}^{C}\right)\boldsymbol{W}_{jj}\right)^{-1}\boldsymbol{W}_{ji}\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{jj}^{A}$$

Make $\boldsymbol{W}_{ji}=\boldsymbol{H}_{ji}^{A}$ and $\boldsymbol{W}_{jj}=\boldsymbol{H}_{jj}^{A}$

$$\boldsymbol{H}_{jj}^{B}=\boldsymbol{H}_{jj}^{A}\boldsymbol{H}_{jj}^{A}\left(\boldsymbol{H}_{ji}^{A}\left(\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{ij}^{C}\right)\boldsymbol{H}_{jj}^{A}\right)^{-1}\boldsymbol{H}_{ji}^{A}\boldsymbol{H}_{ij}^{A}-\boldsymbol{H}_{jj}^{A}$$

THEORETICAL FORMULATION

FRF **uncoupling** - Summary

First formulation

$$\mathbf{H}_{jj}^{B}=\mathbf{H}_{ji}^{A}\mathbf{H}_{ij}^{A}\left(\mathbf{H}_{ji}^{A}\left(\mathbf{H}_{ii}^{A}-\mathbf{H}_{ii}^{C}\right)\mathbf{H}_{ij}^{A}\right)^{-1}\mathbf{H}_{ji}^{A}\mathbf{H}_{ij}^{A}-\mathbf{H}_{jj}^{A}$$

Second formulation

$$\mathbf{H}_{jj}^{B}=\left(\mathbf{H}_{jj}^{A}\left(\mathbf{H}_{jj}^{A}-\mathbf{H}_{jj}^{C}\right)^{-1}-\mathbf{I}_{jj}\right)\mathbf{H}_{jj}^{A}$$

Third formulation

$$\mathbf{H}_{jj}^{B}=\mathbf{H}_{jj}^{A}\,\mathbf{H}_{jj}^{A}\left(\mathbf{H}_{ji}^{A}\left(\mathbf{H}_{ij}^{A}-\mathbf{H}_{ij}^{C}\right)\mathbf{H}_{jj}^{A}\right)^{-1}\mathbf{H}_{ji}^{A}\mathbf{H}_{ij}^{A}-\mathbf{H}_{jj}^{A}$$

SIMULATION STUDIES

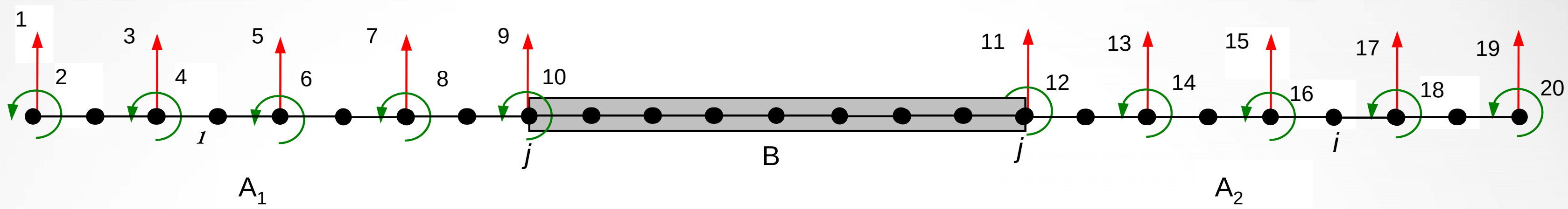


Fig. 2 Test Struture

Beam	Length	Width	Thickness	ν	E	ρ
A_1	300 mm	25 mm	3 mm	0.3	210 GPa	7850 Kg/m^3
B	400 mm	25 mm	6 mm	0.3	210 GPa	7850 Kg/m^3
A_2	300 mm	25 mm	3 mm	0.3	210 GPa	7850 Kg/m^3

Characteristics of the components of the beam

The aim is to characterize component *B* (our "joint"), evaluating $\boldsymbol{H}^B$ assuming that $\boldsymbol{H}^A$ is calculated analytically and $\boldsymbol{H}^C$ is calculated through experiments (simulated, in this case).

Using the finite element method with beam elements with four degrees of freedom.

Choice of the formulation

To simulate the experimental errors, one has imposed a **1%** perturbation in matrix $\boldsymbol{H}^C$ for the **three** formulations.

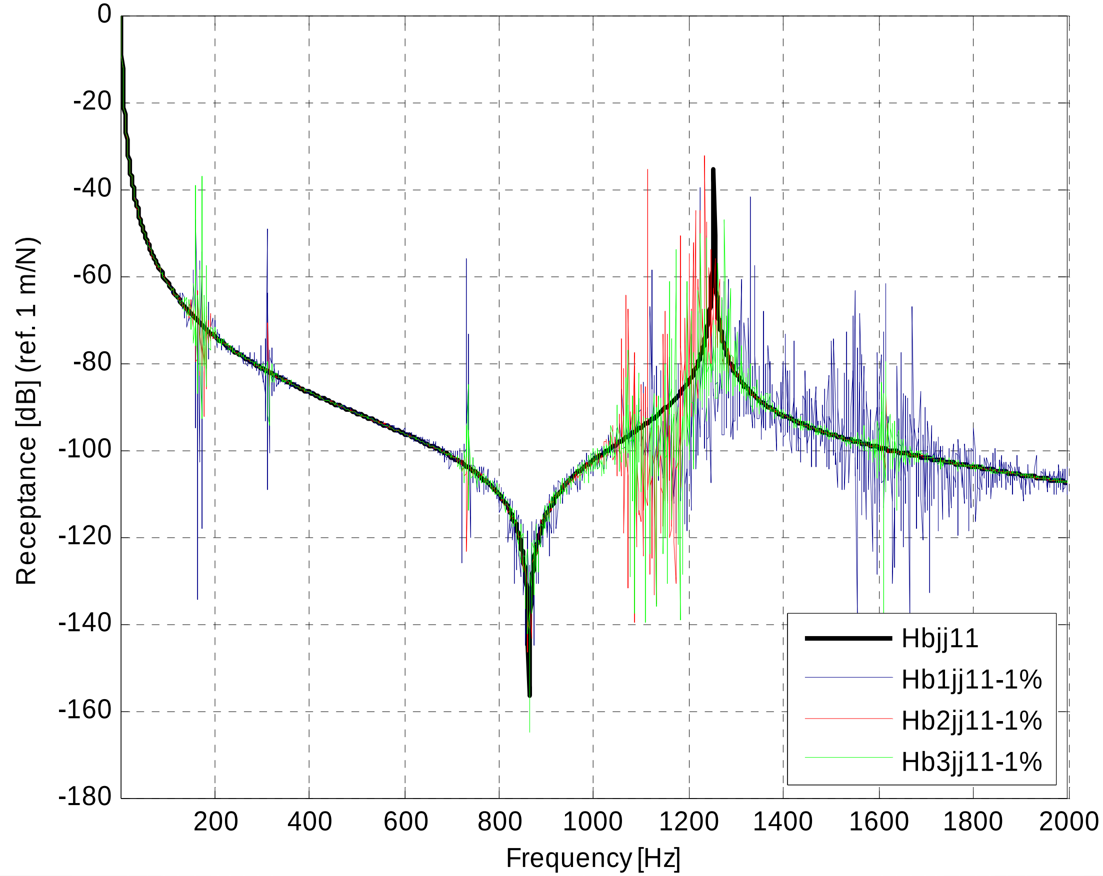


Fig. 3

SIMULATION STUDIES

Choice of the formulation

Error is the module of the differences between the numerically exact response and the response obtained by the three formulations

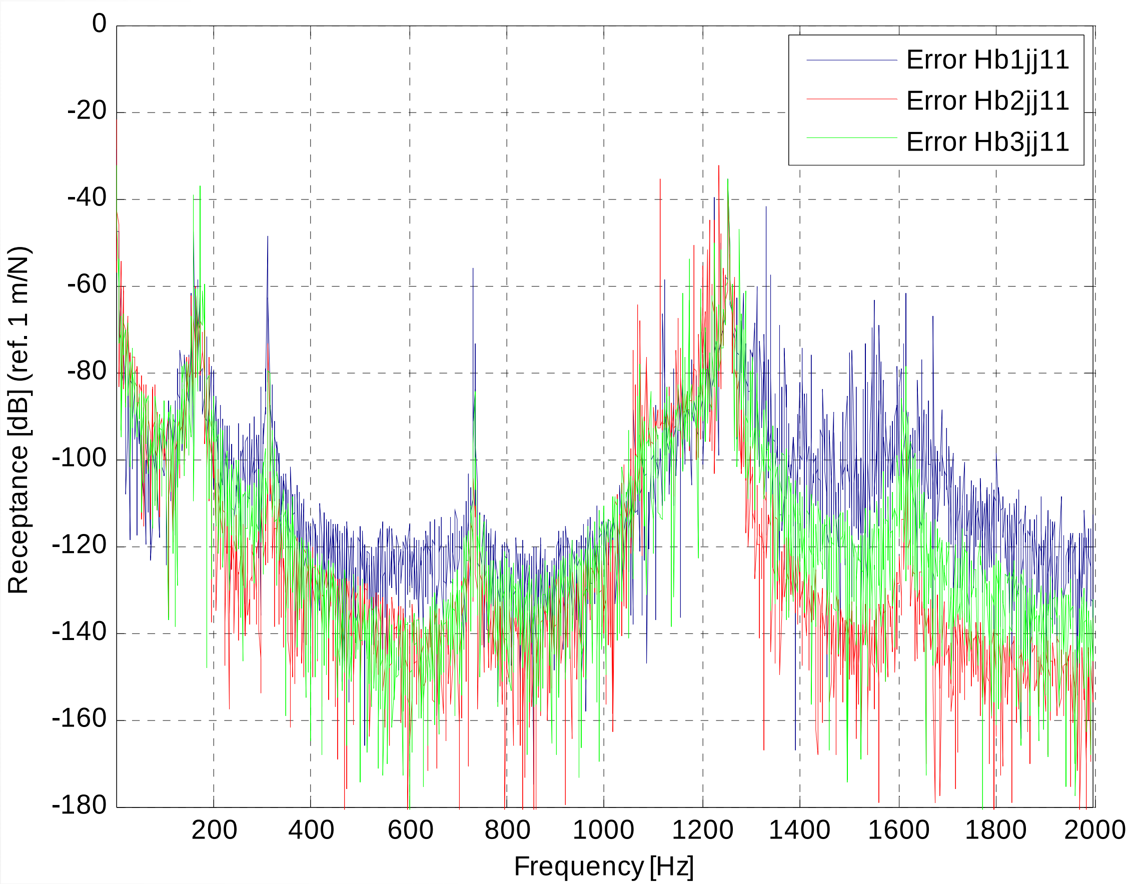
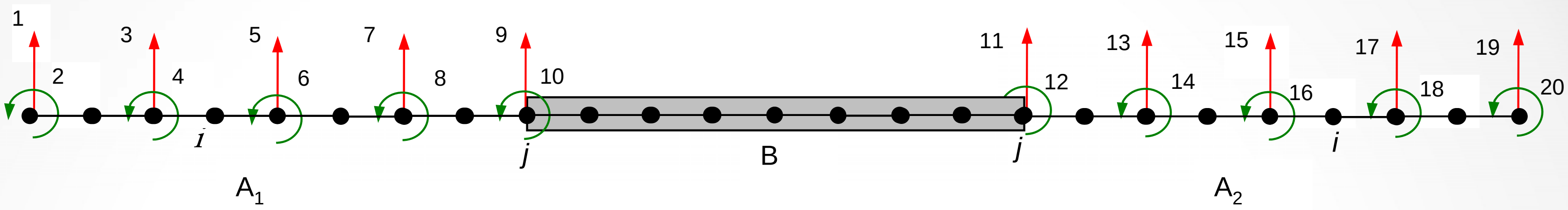


Fig. 4

Errors of *FRFs* H_{11}^{B}

From the three results, the **second formulation** is the one that produces the smallest error throughout the frequency range.

Choice of the formulation



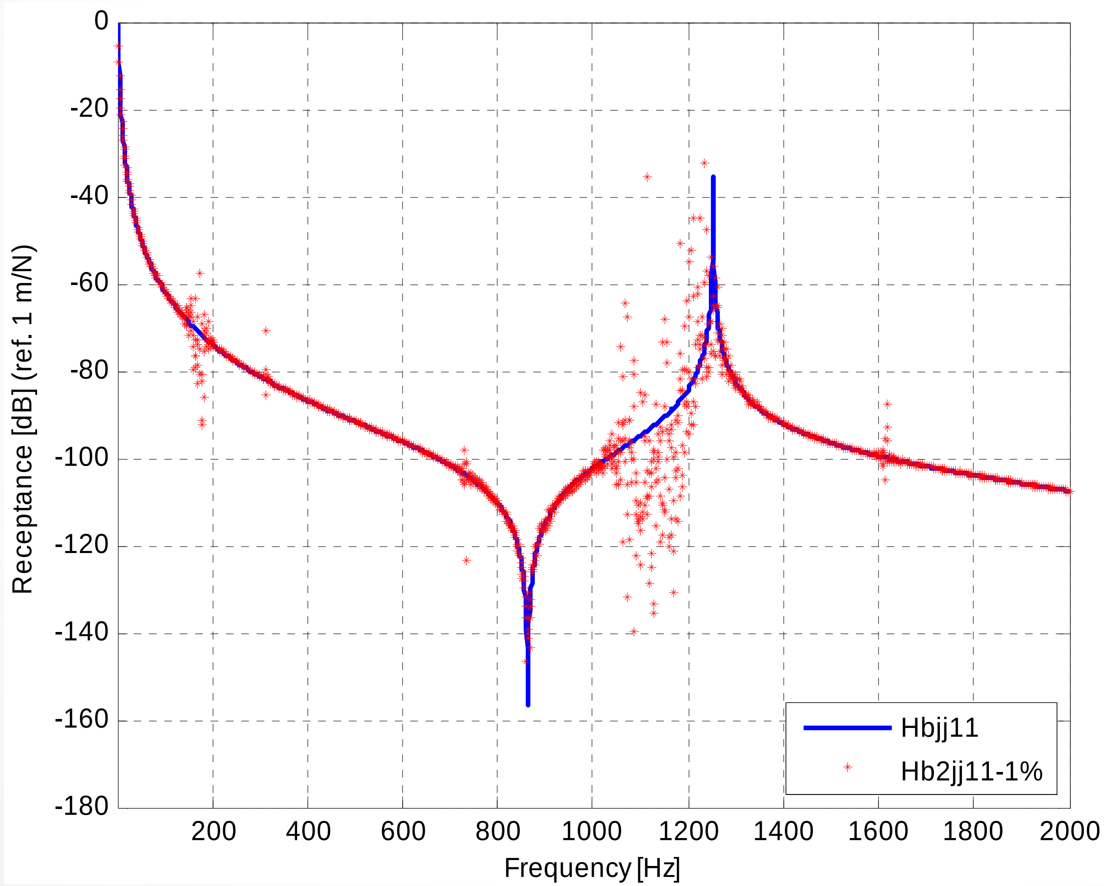


Fig. 5

Represent H_{11}^{B} only for the second formulation, superimposed with the numerically exact response.

SIMULATION STUDIES

Strategies to improve the results

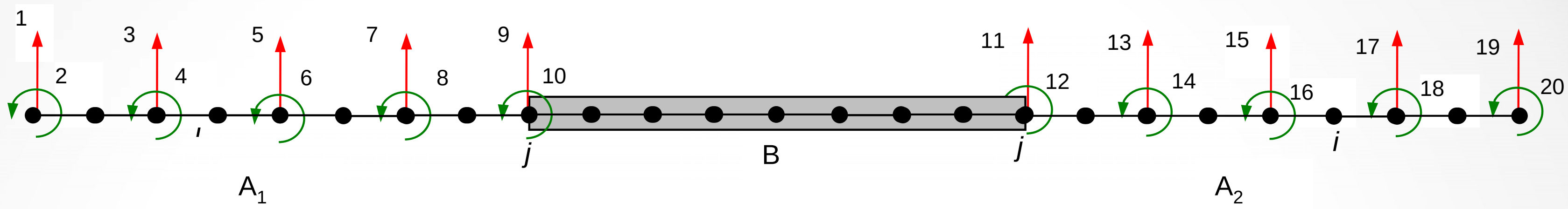
$$\boldsymbol{H}_{jj}^{B} = \left(\boldsymbol{H}_{jj}^{A} \left(\boldsymbol{H}_{jj}^{A} - \boldsymbol{H}_{jj}^{C} \right)^{-1} - \boldsymbol{I}_{jj} \right) \boldsymbol{H}_{jj}^{A}$$

Apparent that the problems certainly arise in the inversion of

$\left(\boldsymbol{H}_{jj}^{A} - \boldsymbol{H}_{jj}^{C} \right)$ namely when this difference is small.

To try to increase this difference one will change our structure by adding point masses, so to change the behavior of sub-structure *A* and *C*, while *B* remains unchanged.

Strategies to improve the results - Adding mass to sub-structure A



A mass of **35 grams** has been added to nodes 1, 3, 5 and 15, 17, 19.

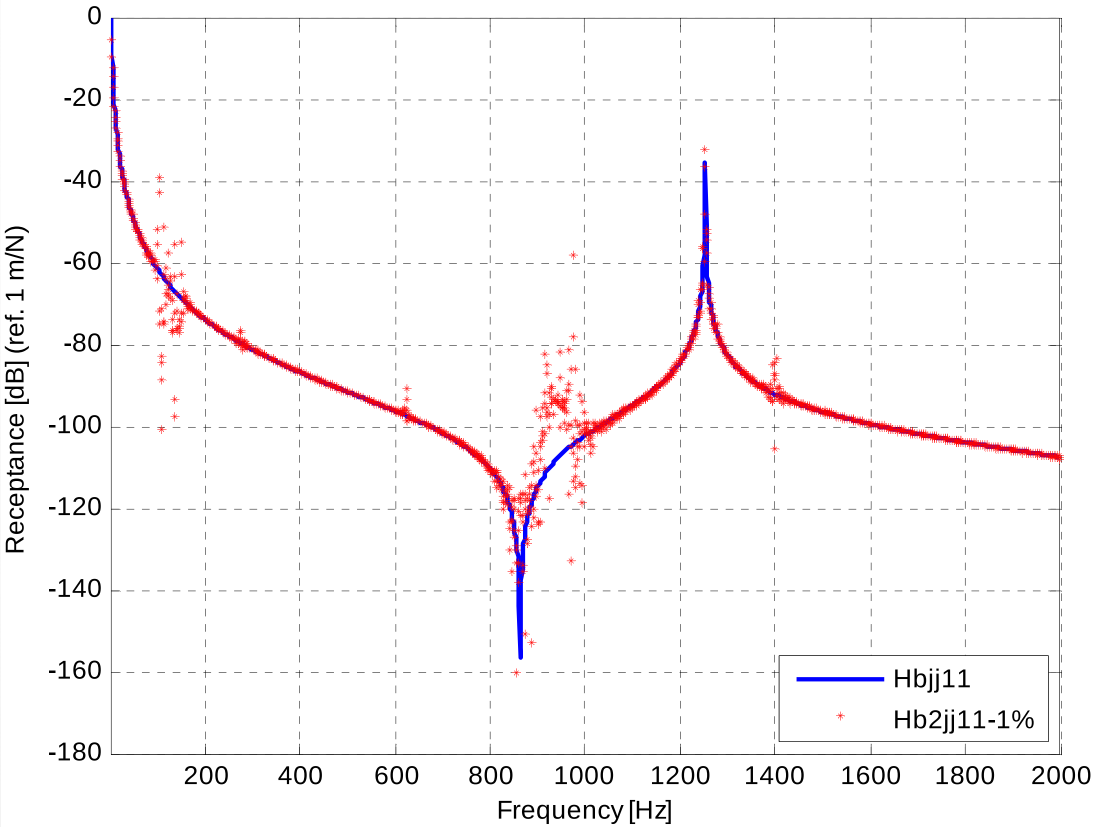
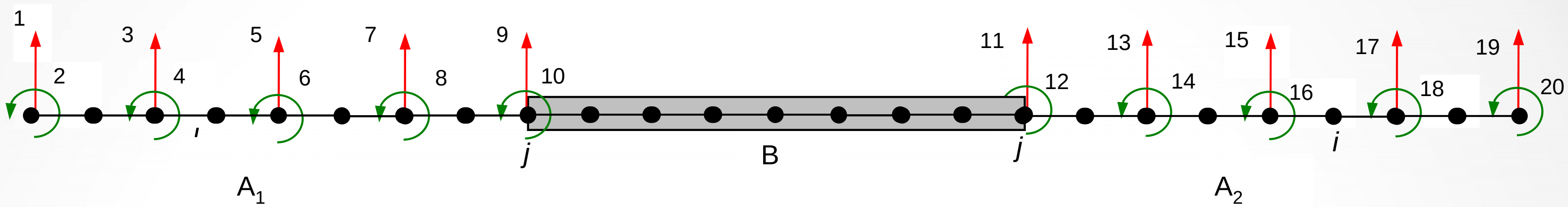


Fig. 6

That the disturbance observed between 1000-1200 Hz moves into the range 800-1000 Hz, generally improving the results

SIMULATION STUDIES

Strategies to improve the results - Adding mass to sub-structure B



A mass of **35 grams** has been added to nodes 9 and 11 in the sub-structure B.

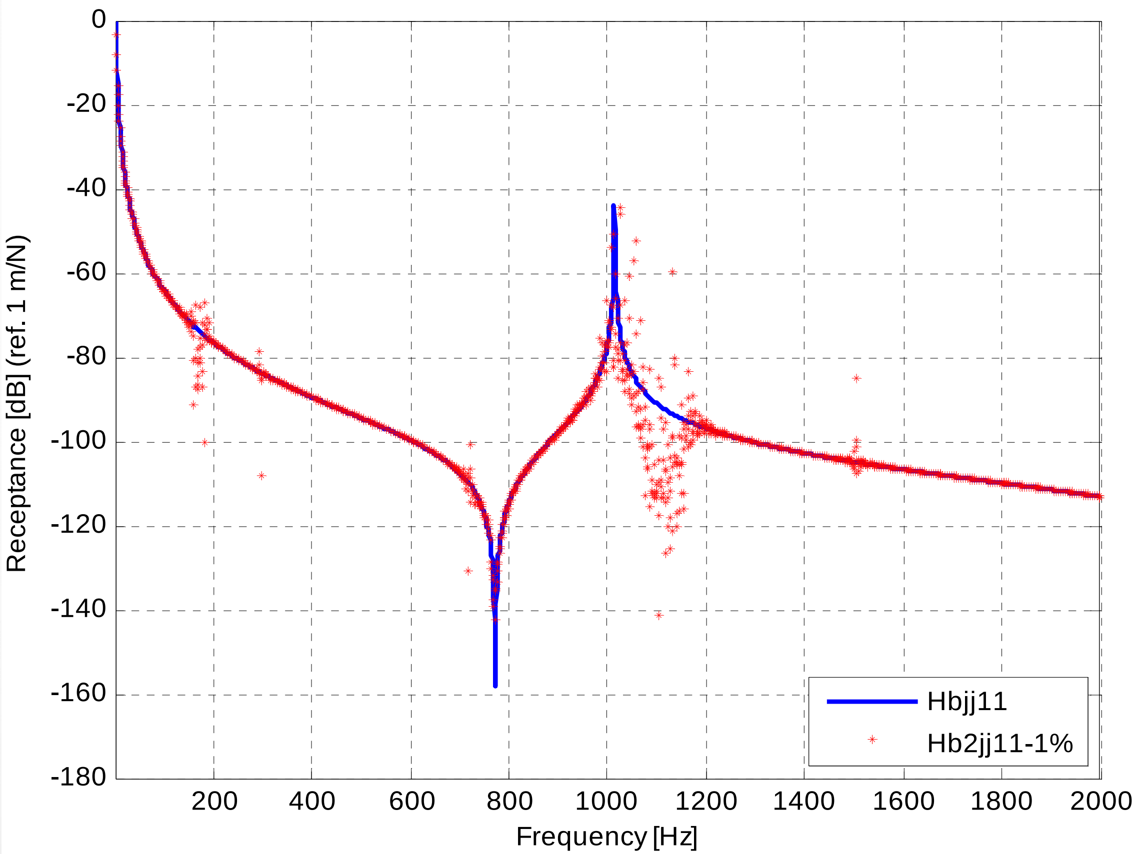


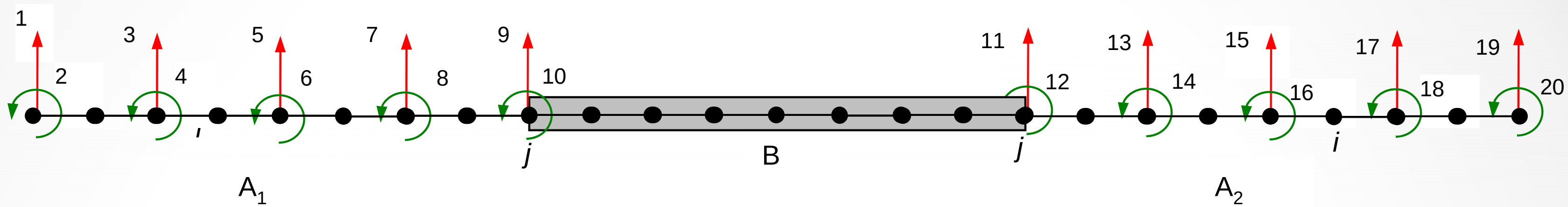
Fig. 7

As sub-structure *B* is altered, its natural frequency changes to the left.

However, the disturbances remain in the area of 1000-1200 Hz and one can conclude that they are caused caused by sub-structure *A*,

SIMULATION STUDIES

Strategies to improve the results - Adding mass to sub-structure B



A mass of **70 grams** has been added to nodes 9 and 11 in the sub-structure B.

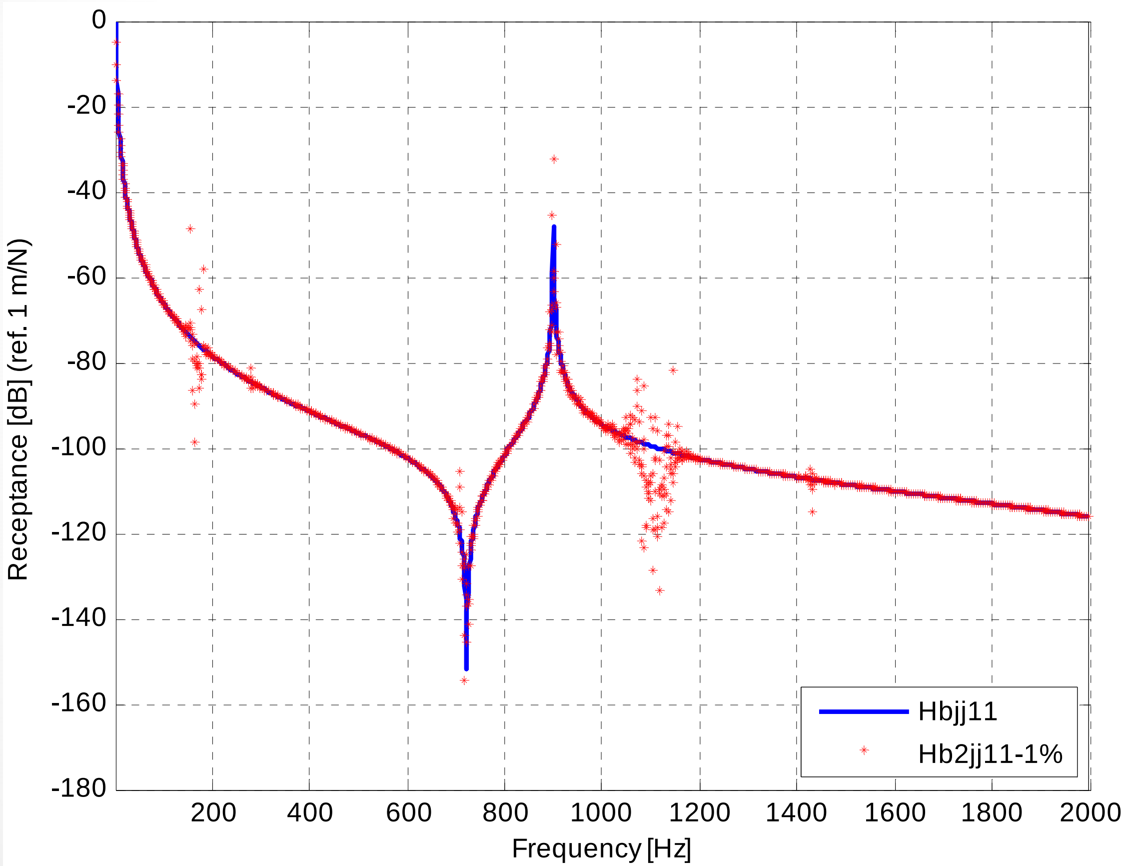


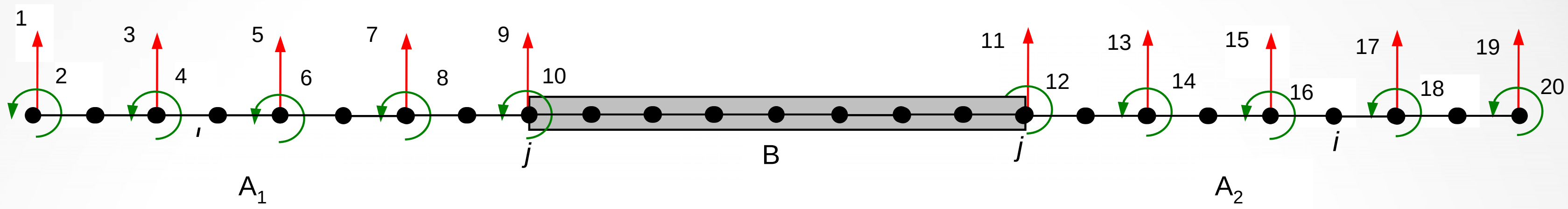
Fig. 8

The results are clearly better.

However, to recover the dynamic response of *B*, one has to **uncouple the added masses**.

SIMULATION STUDIES

Strategies to improve the results - Adding mass to sub-structure B



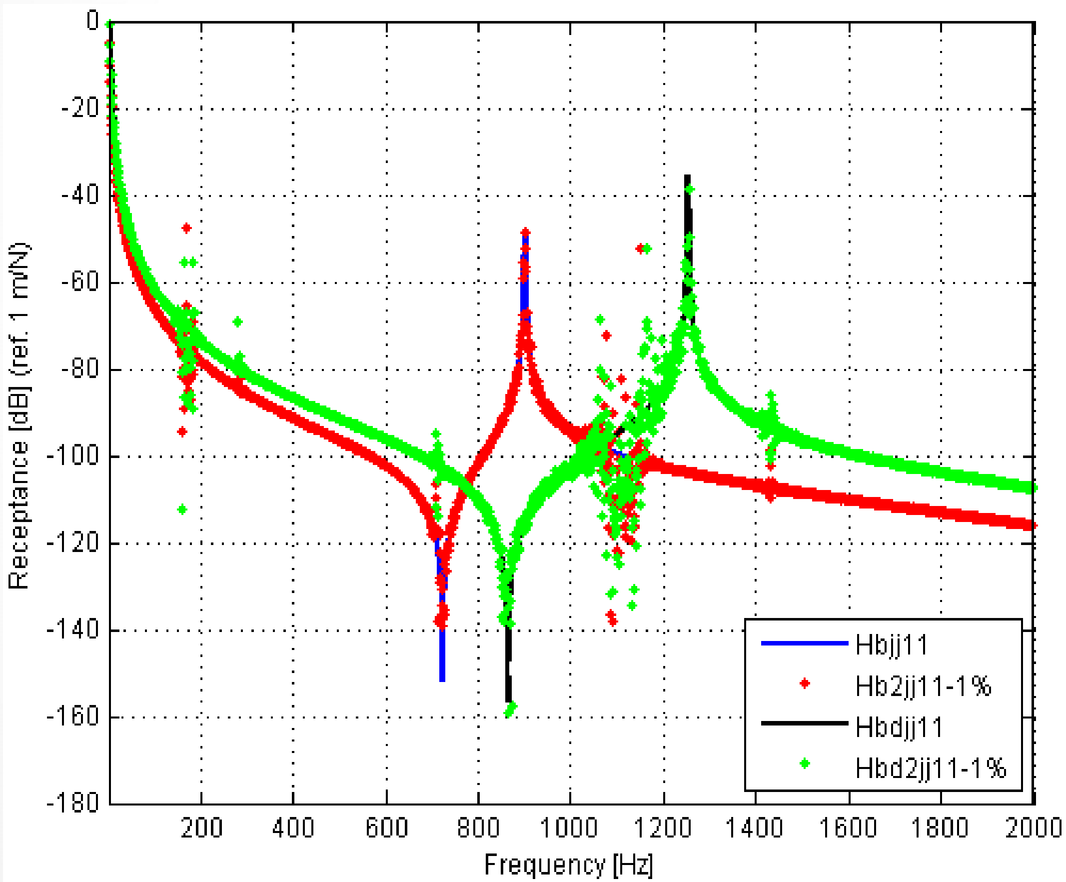
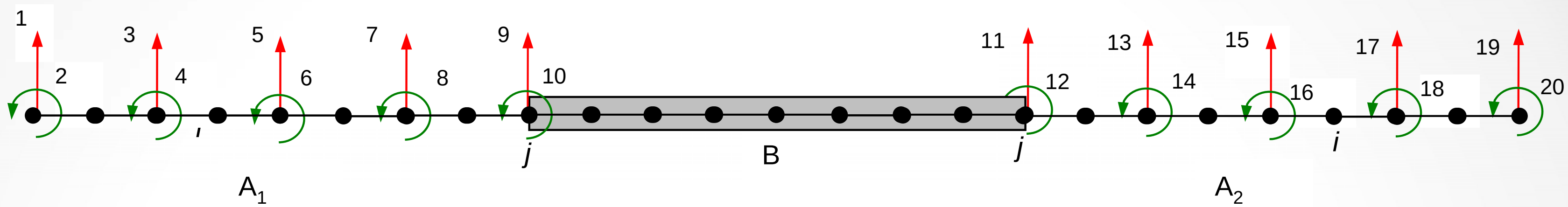


Fig. 9

Uncouple the added masses

Although the results are better than the initial ones (figure 5), they are worse than those of figure 6, when the masses were added to sub-structure A.

Coupling

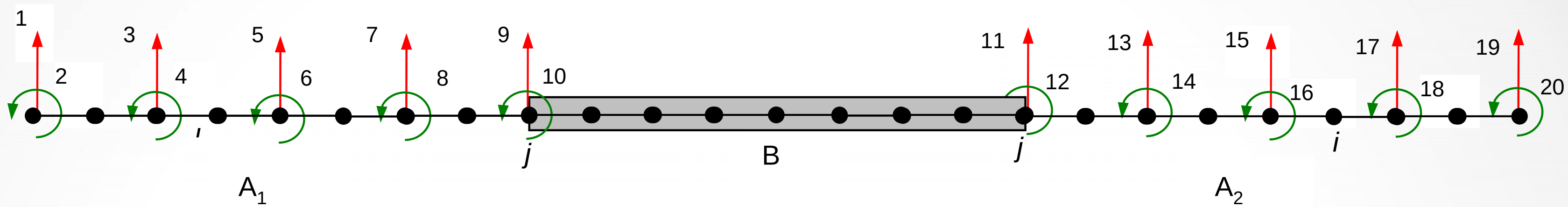


Beam	Length	Width	Thickness	ν	E	ρ
A_1	**400 mm**	25 mm	3 mm	0.3	210 GPa	7850 Kg/m^3
B	400 mm	25 mm	6 mm	0.3	210 GPa	7850 Kg/m^3
A_2	**400 mm**	25 mm	3 mm	0.3	210 GPa	7850 Kg/m^3

One of the main interests of the dynamic characterization of a sub-structure (like a joint) is to be able to predict the dynamic behavior of another structure (or a modified one), possibly a more complex one, inserting (coupling) the identified results from the uncoupling procedure.

Based on the results obtained for sub-structure *B*, a coupling procedure will be undertaken with similar components, two beams *A1* and *A2* but now with a length of **400 mm**.

Coupling



H_{11}^{C} resulting from the coupling of *A1* and *A2* with the sub-structure *B*.

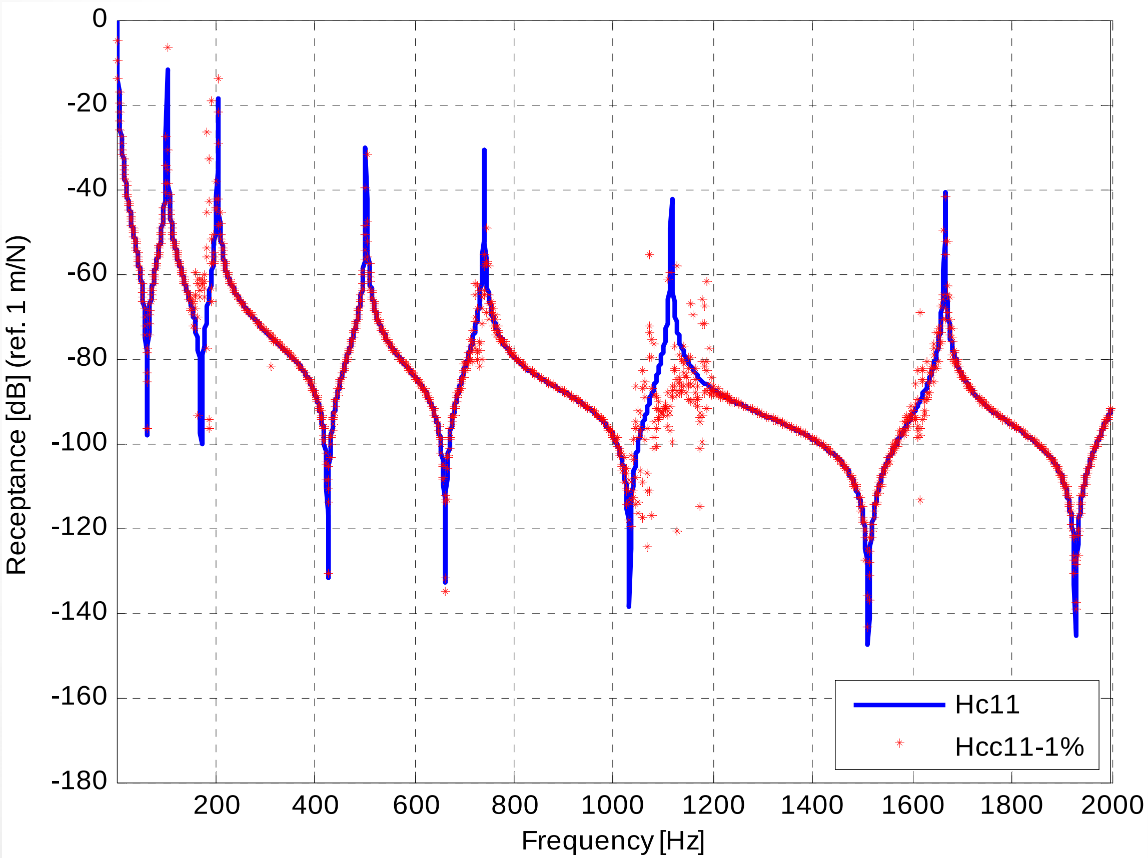
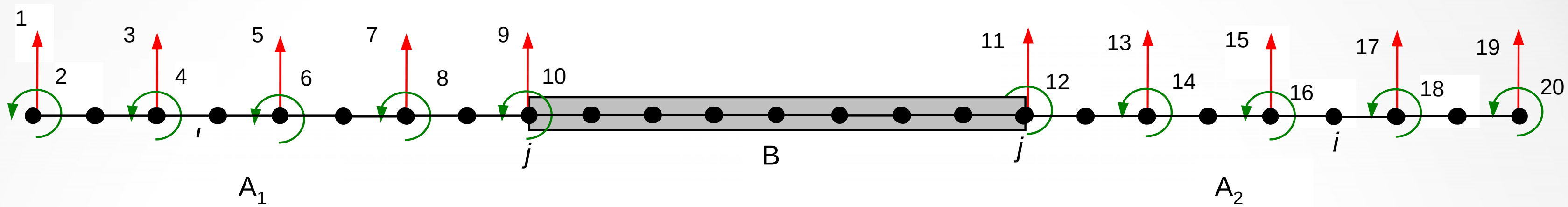


Fig. 10

The disturbance in the area of 1000-1200 Hz remains.

Coupling



H_{11}^{C} resulting from the coupling of *A1* and *A2* with the sub-structure *B*, with the addition of masses in *A*

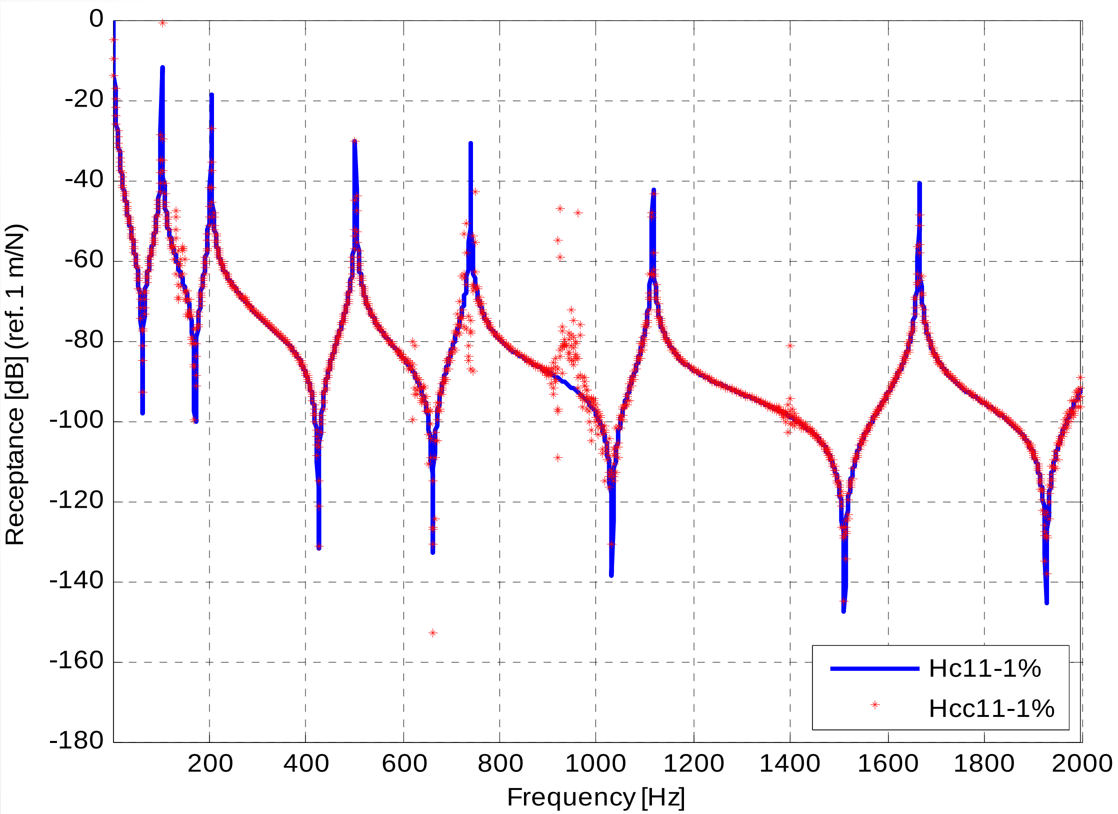


Fig. 11

One can also observe the results of the coupling of B when the masses were added to A, i.e., when using the responses given in figure 6.

CONCLUSIONS

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- The authors have presented three formulations for the uncoupling of sub-structures.

- The formulation that presented the best results requires measurements at the connection points of the structures; unfortunately, this may not always be possible in practice.

- The three formulations revealed to be numerically unstable due to the inversion of difference matrices.

- The improvements have been obtained when adding point masses to the remaining sub-structures other than the one to be characterized. Those added masses move the natural frequencies, allowing to understand the problems that are happening and as already said, improving the results.

- Experimental implementation still has to be further investigated, as the accurate measurement of rotations is quite difficult to obtain.

IMAC XXIX

Thank You!